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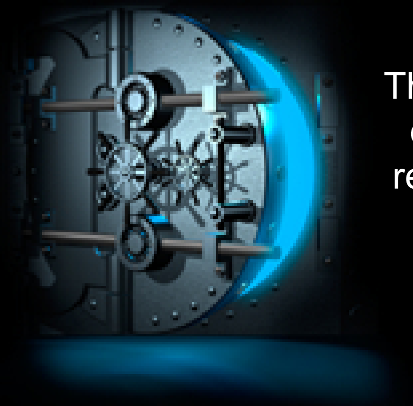

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One critical aspect of space platform maneuverability is the performance of rapid orbital changes. The standard type of steady-state chemical rocket, optimized for firing in the vacuum of space, is regarded as the "base-line" type of propulsion engine against which any new concept should be measured. In the present study of intermittently firing chemical rocket-type engines, such standard type rockets are taken as the reference.
There are still open questions as to whether the specific impulse or propellant flow economy of a chemical propulsion rocket-like system might be improved by some modification of the combustion-and-outflow cycle. The concept of intermittent combustion-and-outflow represents an interesting alternative.
The terminology used to describe such an intermittent process has been confusing in the past, with termslike detonative propulsion, explosive propulsion and pulsejet propulsion all having been used. One of the tasks of this project is necessarily to clarify the differences implied in these terms.
This Final Report contains solutions to sample problems addressing combustion

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EXECUTIVE SUMMARY

This report contains documentation of the efforts of Princeton Combustion Research Laboratories, Inc. in the performance of the Statement of Work of Contract F49620-82-C-0042. Unfortunately, the objectives of this research study, as outlined in the Statement of Work, were not satisfied.

We have attempted in this Final Report to provide AFOSR with the details of a literature review, solutions to sample problems (quasi-steady analysis), and a recommended approach for non-steady geometric optimization and associated performance efficiencies.

During the period of performance of this effort, the following personnel were associated with the research effort: Dr. Martin Summerfield, Dr. Claudio Bruno, Dr. Moshe Ben-Reuven, and Dr. Hsing Tseng. During January 1982, PCRL participated in a workshop on Orbit-Raising and Maneuvering Propulsion, held in Orlando, Florida.

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STATEMENT OF WORK

TASK I. Begin the analysis of unsteady explosions and detonation in vacuo, including: study of characteristic times; study of geometry; and comparisons between explosions and detonations.

TASK II. Extend the analysis of Task I to the case in which an inert substance is accelerated, including: study of geometry; and analysis of the energy exchange between energy release products and inerts.

TASK III. Study the effect of the energy release in the presence of material boundaries, including: thrust and specific impulse evaluation; performance criteria; and energy deposition at the boundaries.

TASK IV. Study of the effect of the initiation process on the energy-release process, including: laser beam initiation and chemical initiation.

I. Background of the Present Study

It is visualized that, in the future, perhaps about the year 2000 A.D., the U.S. Air Force will be engaged in the development of very large space platforms for military purposes. Large rockets will probably be employed to raise such platforms in component parts into low earth orbit; and then smaller propulsion units will be employed for maneuvering the components into position for assembly and for raising the assembled orbiting vehicle from the initial low orbit to some higher orbit. After placement in the initially desired orbit, another propulsion system should be available on board for subsequent changes in orbit, higher or lower, and possibly for other purposes as well.

It is far too early in this kind of program to define the specific military objectives of such large space vehicles, the probable mass of such a vehicle, the velocity corrections that might be desired, the response times of the on-board propulsion systems, the accelerations that might be required, the degree of "fine-tuning" that may be needed, and so forth. At this stage of the study, emphasis is being given to the specific impulse or the mass consumption per unit impulse, of the propulsion system. However, it is highly probable that consideration will be given by designers, when such vehicles are laid out for specific missions in the future, to such additional characteristics as those mentioned, and to questions of energy consumption per unit impulse, the mass of the thrust engine, the manner of thrust control, and so on. An additional driving factor in making the selection of an engine will surely be the question of the date when it is needed and how soon the space vehicle design has to be frozen; the designer will have to choose from those types that are likely to be available in practical, proven form when the design is decided upon. For the purposes of this study, all such questions, despite their supreme importance for the development of a military space veicle in a particular military context, have been set aside at the request of the Air Force Office of Scientific Research.

We wish to say parenthetically, in this introduction, that we visualize some contradiction between the time frame suggested (the year 2000) and the deliberate setting aside of all design and practical questions other than efficiency, specific impulse, and energy consumption. Nevertheless, for this short study, we have focused on those issues alone. Two decades is a very short time, in our experience, for the complete cycle, from researching a novel concept of propulsion, to proving the concept in laboratory type hardware in small scale, to developing a full scale experimental engine, to developing a prototype, and then to freezing the final design. The short time suggested for this study almost demands that a selection be made from among concepts that rest today on proven principles. The AFOSR program enbraces a variety of propulsion concepts - chemical rockets, electrical ion beam or plasma propulsion, laser beam heated gas jets, electromagnetic accelerators, etc. The short time deadline suggested for this study seems to dictate a conservative choice among them.

The standard type of steady-state chemical rocket, optimized for firing in the vacuum of space, is regarded as the "base-line" type of propulsion engine against which any new concept should be measured. In the present study of intermittently firing chemical rocket-type engines, such standard type rockets are taken as the reference.

A second parenthetic remark is offered here on the question of choice of orbit-raising type of propulsion system for the future. It seems probable that the propulsion system to be used for orbit adjustment of a military space vehicle will also be called upon to maneuver the vehicle to avoid hostile weapons or to position the vehicle quickly from one battle position to another. It would seem, therefore, that preference would be placed on propulsion concepts that can offer substantial vehicle accelerations economically as against concepts that are limited to small or moderate thrust levels and low vehicle accelerations. Such considerations tend to favor the chemical rocket, for example, as against the electric ion accelerator. Of course there may be other practical types of moderate-to-high thrust propulsion engines, but the chemical rocket looks attractive for military type space vehicles.

The rationale for the present study is that a chemical propulsion rocket-like system is probably a first-choice system for military space vehicle application in the next two decades, but there is still an open questionas to whether the specific impulse or propellant flow economy might be improved by some modification of the combustion-and-outflow cycle. The concept of intermittent combustion-and-outflow represents an interesting alternative. The terminology used to describe such an intermittent process has been confusing in the past, with terms like detonative propulsion, explosive propulsion, and pulsejet propulsion all having been used. One of the tasks of this project is necessarily to clarify the differences implied in these terms. Another issue that tends to arise is, what is meant by the efficiency of a propulsion system? The practical answer is simply that the efficiency is inverse to the specific propellant consumption; that seems obvious. Nevertheless, the literature contains analyses that indicate that somehow the efficiency depends on whether the emerging jet is moving slow or fast with respsct to a chosen frame of reference, e.g., the earth or the fixed stars. A discussion of this question is contained below.

II. Modes of Intermittent Propulsion

To describe the various modes of intermittent rocket-like propulsion, it is useful to consider the component processes and their characteristics. Within the scope of the combustion-based propulsion systems being considered, there are three basic component processes: chamber-filling, combustion, and outflow. In a steady-state rocket of liquid propellant type, these three component processes function steadily and simultaneously, and their respective characteristics are accurately described by steady-state theory. The same is true of a turbojet, and it is essentially true of a solid propellant rocket of the usual long-burning type. An air-breathing pulsejet, however, requires a different form of analysis. Approximate analyses of the intermittent pulsejet have been published (see the literature study given below); indeed, it is with such non-steady approximate treatments in mind, that the present analysis of the non-steady rocket has been formulated.

The three component processes listed above tend to overlap in time, in any real propulsion system, but for the purposes of analysis, we may treat them as non-overlapping. This leads to important simplifications, but admittedly, it leaves unsettled the essential question of whether the resulting theoretical specific impulse is accurate. Since we do not really expect to find large increases in specific impulse in this investigation of non-steady rocket-like propulsion systems, we have to concede in adopting such simplifications that this short study may not be the final answer. This question is discussed further after the results of the non-steady analysis are deduced and compared with steady-state analysis, in the sections below.

For each of the propulsion configurations considered in this study, it has been assumed that the combustible substance (propellant) and any additional inert working fluid are completely injected into the combustion chamber before the commencement of combustion and before the consequent pressure rise. In practical propulsion engines, this may not be so. In fact, in the interest of having the engine produce as many pulses per unit time possible, consistent with the highest level of specific impulse, some overlap is tolerated and, in fact, desired. This is true in the pulsejet engine designed by E. Schmidt, used by the German side for the V-1 Flying Bomb in WW II. The loss of performance due to such overlap is outside the scope of this brief study, and so the assumption has been made that no energy is expended in injecting the combustible charge and the working fluid into the combustion chamber.

Next is the question of the mode of combustion of the charge. For an intermitent engine, the appropriate mode of combustion may be called explosive, but this general term -- in combustion science -- can be broken down into three separate types, one, homogeneous exothermic fast reaction, two, rapid deflagration from one or more ignition points and three, detonative reaction, which involves propagation and reflection of strong shock waves within the chamber. Thermodynamically, if the chamber remains adiabatically sealed, the final combustion temperature is the same, for all three modes, if the reactants are the same. Thus, in actual operation, there is no performance distinction between any of the modes, as long as the particular combustion reaction is rapid enough to justify the assumption that the explosion time is very short compared with time for the subsequent exhaust outflow process. Thus, we refrain from using the term "detonative propulsion" when the more all-inclusive term "explosive propulsion" is thermodynamically equivalent and when the performance analysis does not distinguish between the three modes of combustion described above.

Finally, there is the question of the outflow process. In an intermittent engine, this is usually a non-steady process, by which is meant that the characteristic time for exhaust of the fluid from the combustion chamber is comparable to the wave reverberation time of the exhaust duct. This would usually forbid the treatment of the exhaust process by steady-state analysis, even with time-varying conditions at the ends of the nozzle or duct. Nevertheless, a truly non-steady analysis is so costly in time (involving inevitably step-by-step treatment of wave motions and computerized computations), even if performed in one dimension, notwithstanding the essentially axisymmetric two-dimensional character of a typical engine configuration, that it could not be performed within this brief study. Without really knowing how great an error in the performance analysis is created, the approximation of quasi-steadi outflow was adopted in the treatments contained herein.

Thus, for the purpose of this brief analysis, the simplifications adopted were that: (1) no energy is expended or chargd against the jet specific impulse for the injection process; (2) the combustion process, whether detonative, deflagrative, or homogeneous, has a characteristic time that is short compared with the outflow time, and that it is adiabatic and perfectly complete; and (3) the outflow time is long compared with the wave transmission time within the exhaust duct. Unfortunately, the assumption that these processes do not overlap and that they are quasi-steady as described, while simpifying the analyses, leaves unsettled the essential question of whether a modest performance gain might be achieved by some form of non-steady rocket-like propulsion.

III. A Figure of Merit: Efficiency or Specific Impulse

In the analysis presented herein, the figure of merit for the performance of a particular engine cycle (steady-state or intermittent or other) is chosen to be the well-known specific impulse, that is, the impulse per unit mass of propellant consumed. Inasmuch as there is also some interest in a specific impulse defined as the impulse delivered by the engine per unit of total mass consumed, i.e., the propellant mass plus any associated inert mass driven out by the propellant gas, that type of specific impulse is computed also and presented herein as well. All comparisons, therefore, are based on the computed specific impulse values.

In order to have a valid basis for comparison, certain uniform assumptions were made in the analysis. Thus, the flow was assumed to be frictionless, the processes of combustion and outflow were taken to the adiabatic, the expansions were taken to be isentropic, the explosion pressures were specified in each case, the gas properties were taken to be constant, independent of temperature and pressure, and equal for all cases, etc. The particular equalizing assumptions are give in the sections of analysis.

The question of whether the overall efficiency, defined in several prominent books on jet and rocket propulsion, should be taken as a figure of merit as an alternative to specific impulse was particularly troublesome. The equation for the propulsion efficiency given by several book authors is:

$$\eta_p = \frac{2u/c}{1+(u/c)^2}$$

It is derived from the notion that the useful work done by the jet engine is that expanded in driving the vehicle at its velocity of motion, and that the wasted energy is the absolute kinetic energy of the jet gases based on the net velocity, i.e., the jet velocity with respect to the vehicle minus the vehicle velocity. In this definition, the wasted kinetic energy of the jet becomes zero when the jet velocity and the vehicle velocity are equal in magnitude, i.e., when the jet gases emerge at rest, and then the propulsive efficiency becomes one. The propulsion efficiency defined in this way is then multiplied by the thermal efficiency of the jet or rocket engine to arrive at a so-called overall efficiency.

This same formula can be derived from the notion that the input energy is the fraction of propellant heat of combustion that is converted to jet kinetic energy (measured on a firing test stand) augmented by the kinetic energy of the propellant as it is being carried by the vehicle in flight; this gives the propulsion efficiency which, when multiplied by the thermal efficiency, gives the overall efficiency.

Either way, the definition requires an unambiguous measure of the velocity of the vehicle or the kinetic energy of the vehicle and its propellant content. Unfortunately, such an unambiguous "absolute" measure does not exist. The vehicle velocity depends on the frame of reference in space, and there is no unique frame to choose. It is appealing to choose the earth as a frame at rest, but the earth rotates and moves in its orbit, and the solar system is moving with respect to the fixed stars; so, the "rest frame" is not identifiable. Any desired value of propulsion efficiency can be computed, depending on the reference frame chosen.

We have tried to develop alternative definitions of the so-called propulsive efficiency, but they all depend on an arbitrary choice of the frame of reference for measuring the vehicle kinetic energy. As a consequence, we have abandoned the overall efficiency or the propulsive efficiency as a figure of merit. To the best of our knowledge, no other engineering organization has ever adopted the propulsive efficiency or the overall efficiency as a figure of merit as a guide to the selection of a jet propulsion cycle or as a guide to the search for cycle improvements. Consequently, in this report, only the specific impulse (or its inverse, specific propellant consumption), is used as a figure of merit for comparing different hypothetical steady-state and non-steady rocket cycles.

IV. Literature Review

To analyze the history of intermittent propulsion is instructive, since many of the questions that apply to space thrusters were encountered in the course of developing other intermittent propulsive devices. Generally speaking, progress in this field developed along four successive directions: pulsed engines, where compression of the reactants was (partly) achieved by valves; wave engines, using wave motion for compression; detonation wave engines, where a detonation wave accelerates the gas; and engines using high explosive energy release.

Foa (1960), in his accuont of the history of nonsteady-flow thrust generators, points out that the idea of intermittent combustion preceded that of the steady flow combustion common to present-day gas turbines (Amsler, 1900; Diedrich, 1948; Barbezat, 1909; Lorin, 1908; Stodola, 1945). In particular, Lorin (1908) proposed a jet engine in which combustible gas, compressed in a combustion chamber by a reciprocating compressor, explodes and expands periodically in a nozzle through an alternating valve. Marconnet (1910) proposed a "reacteur-pulsateur" device which contains all the elements of the German buzz-bombs, i.e., a one-way valve admitting air, a fuel injector and a long exhaust tube (Fig. 1).

Schmidt (1930, 1931) obtained two patents which were the starting point for the V-1 pulsejet of WWII. This engine was composed essentially of an appropriately shaped duct, with a row of one-way air valves at the air inlet (Fig. 2). Fuel was sprayed continuously past the row of valves, resulting in the relatively poor specific impulse and range of this device. Although the V-1 was operational for a number of years, the detailed explanation of how a V-1 pulsejet work lagged considerably its development. Stipa (1938) also had analyzed pulsating combustion in a duct under simple assumptions, but was unaware of the V-1 development effort in Germany, and never transformed his ideas into a practical device.

At the end of WWII, sufficient material had been made available to the Allied Forces to analyze the V-1 performance and to begin to understand its operational principles (Anon., 1944a, 1944b). Considerable uncertainty, however, surrounded the role of the long tailpipe. Ts'ien (1946) analyzed the V-1 performance using a simple zero-dimensional (i.e., lumped parameters) thermodynamic approach and made a comparison with the experimental data obtained from running the V-1 pulsejet. His theoretical analysis indicated that for the best specific fuel comsumption the flight Mach number should be either <<1 or >1. The region of worst performance, however, tends to become more and more narrow as the ratio between pressure at the end of combustion (p_3) and pressure at the beginning (p_2) is increased. For p_3/p_2 of the order of 10 the specific impulse becomes constant and of the order of 1200 s. For p_3/p_2 of the order of 5, as that actually used in the V-1 engine, the specific impulse is roughly 700 s, low with respect to a conventional turbojet of the same period. This analysis agrees qualitatively with the experimental data.

Much of the work that followed was directed toward a better understanding of the flow field and its periodical time-dependent evolution inside the engine, and toward ways of eliminating the inlet valving by using pure fluid dynamics, i.e., intermittent compression of the air/fuel mixture by appropriately-timed internal wave motion. A good review of these aspects of the air-breathing pulsejet can be found in Foa (1960), Chapter 15. Fig. 3, showing a sketch of a typical wave engine, is taken from Foa (1960). His work (1954, 1959) and the work of Rudinger (1952), MacDonald (1946) and others made clear that a good portion of the combustion air, as well as the main propulsive fluid, is supplied by the backflow from the exhaust outlet into the combustion hamber of the engine. Far from being undesirable, this flow obviously increases the thrust, provided that its net average axial inflow velocity is less than the corresponding average jet exhaust velocity. The explanation of this phenomenon lies in the higher density of the column of gas in the tail pipe: backflow provides a heavy column of cold external air in place of a lighter column of hot combustion products, and it can be shown that, to produce a given impulse $F\Delta t$ with a given heat release, it is more efficient to have within the product $m\Delta v$ a large m rather than a large Δv. In essense, what the pulsejet does is to compress and expel a slug of heavier gas by the piston-like thrust of hotter and lighter combustion gases.

A serious question in wave engines was how to reduce upstream momentum spillage during operation. Great ingenuity went into devices that would allow air to enter the combustion chamber but make difficult its exit upstream. The Bertin rectifier is an example of such "aerodynamic" valve. Wave engines built at SNECMA even had their front air inlet turned 180° backwards, transforming momentum spillage into thrust, at the expense of inlet efficiency.

Work in wave engines continued throughout the 1950's. Zwicky (1951) pointed out the importance of timing in such propulsion and suggested the use of the "rocket pulse" engine, where part of the oxidizer in the rocket would be supplied by intermittent overexpansion inside the nozzle while the vehicle was still within the atmosphere. In seeing the possibility of propellant self-pumping he was not anticipating its negative aspects in what was going to be studied as "pogo instability" in rocket engines a decade later. Work on conceptual derivations from the V-1, called variously valveless aeropulse, pulsejet, or thermojet, was continued by MacDonald at New York University (1945), and ended there approximately in 1951 with Macdonald's death, while Foa kept investigating valveless pulsejets and so-called wave engines until the late 1950's.

By the end of the 1950's the understanding of the V-1 as a device breathing air from the front and from the exhaust duct was complete. Abou the same time, and motivated by exploring the limits of pulsejet performance, Nicholls and coworkers at the University of Michigan conducted a series of investigations of intermittent detonation as a propulsive device, using hydrogen and acetylene with either air or oxygen. The experimental set-up consisted of a detonation tube, 1" in diameter and 6' long, into which air (or oxygen) and fuel were introduced at one end. A spark plug ignited the mixture at about 10" downstream of the fuel injection section. The entire assembly was mounted so as to measure thrust versus time by means of strain gauges. A zero-dimensional analysis was used to predict thrust and specific impulse, under the assumption that a Chapman-Jouguet detonation develops immediately starting at the closed end of the tube.

The experimental results obtained (Nicholls et al, 1957) indicated an instantaneous thrust per unit area of the order of 2,000 lb/ft^2 (94,000 Pa) for C_2H_2/O_2 mixtures, when the pre-detonation pressure was 1 atm. abs. and of about 500 lt/ft^2 (31,000 Pa) for H_2/O_2 mixtures. Concurrently the specific impulse was lower than for conventional rockets, i.e., 120 to 160 s for the two fuels investigated. These results are not surprising since the expansion products following the detonation wave are expelled from the exhaust outlet at high temperatures carrying unutilized thermal energy, and since the geometry of the pipe does not allow for gas expansion. Moreover the low values of instantaneous thrust indicate that the mixture was not uniformly stoichiometric. A sharp dependence on detonation frequency was also found, with a maximum at 0(10)Hz, the reason being that at low frequency the mixture spills out before ignition, and at high frequencies there is no time to fill the tube completely with fresh reactants.

The work of Nicholls and his collaborators was continued in the direction of spinning detonation wave rocket engines (Nicholls and Cullen, 1964). The problems encountered in detonating a two-phase mixture of gaseous oxidizer and liquid fuel causes the interest in the spinning detonation rocket engine to fade in the U.S., while Edwards (1977) in England was eventually successful.

Approximately a decade later there was a resurgence of interest in propulsion by detonating using solid explosives. This was partly due to Project Outgrowth, whose purpose was to generate new advanced propulsion concepts (Mead, 1982). Gross (1970) conducted a series of computations to determine preformance of explosive propulsion of steel and aluminum plates. He used one-dimensional hydrodynamic equations, and either a gamma law, or a Jones-Wilkins-Lee equation of state for the gas. The explosives were Composition B (Grade A), HMX, NM, El-506-C, and MOX-1, an aluminized explosive. The configuration consisted generally of a sheet of explosive attached to a flat plate simulating the propelled vehicle; however, in two of the computations the explosive was modeled as detonating at certain distance from the plate. In both cases the detonation as unconfined. The results of the computations prdicted a specific impulse ranging from 200 s (for Composition B, Grade A explosive detonated at a distance from the plate equal to the explosive thickness) to 321 s (for MOX-1 explosive attached back-to-back to the steel flat plate).

On the basis of these results the Air Force Rocket Propulsion Laboratory recommended further work on this concept (Mead, 1972, p. I-68). In the course of his investigation, Gross was made aware of experimental work by Bestgen and Nunn (1972) at the Air Force Rocket Propulsion Laboratory. They conducted a theoretical and experimental investigation of the performance of a propulsion system using plasticized explosive. Their theory is one-dimensional and, like Gross', makes the usual Gurney approximation (Gross, 1972, p. 53) for the gas velocity following the explosion. Three configurations were examined: free-expansion, in which the explosive is detonated on the surface of the vehicle and the gases are free to expand, partially confined, in which the detonation gases accelerate an inert mass, and totally confined detonations in which the detonation gases expand against an immovable inert mass.

The results of the calculations nidicated that free expansion delivers a specific impulse comparable to that of a conventional solid propellant rocket engine; partial confinement using a recoil mass, (or stacking explosive charges between materials) increases propulsion performance, but not as much as adding a mass of explosive equal to the recoil mass; and a totally confined detonation yields the best specific impulse, which increases asymptotically to about 3000 s as the ratio (m/M) = (explosive mass/vehicle mass) goes to zero. Experiments were also performed with steel cubes and bars propelled by Composition B explosive, and measurements taken with high speed film, photoelectric timing wires and X-ray cameras. The tests produced a certain amount of scattering, showing however a reasonable agreement between specific impulse predictions and measurements for totally confined explosions, while for unconfined explosions the data did not show a reliable trend.

Based on these results Bestgen and Nunn advised further testing and better experimental technqiues, and cautioned about the danger of vehicle damage for large enough m/M ratios. Comparison of their test results with the theoretical results, of Gross (which were limited to unconfined explosions) reveals a qualitatively good agreement; more important is the high Isp predicted and measured for (m/M) or order 10, for the totally confined detination case.

Seemingly unaware of the earlier work in this field, Varsi and Back (1973), Varsi, Back, and Dowler (1973), Back and Varsi (1974), Kim, Back and Varsi (1974, 1977), Back, Dowler and Varsi (1983) at the Jet Propulsion Laoratory, investigated detonation propulsion for spacecraft designed to penetrate dense planetary atmospheres, such as Jupiter's or Venus'. Conventional rocket engines operating in a steady-state mode yield poor performance when external pressure is high, since specific impulse and thrust depend on the pressure ratio. Intermittent detonations of sufficiently short duration would, as envisaged by the authors, operate at pressures in the 20 to 200 kbar range which tends to overcome the effect of outside pressure (down to about 0.1 kbar deep inside Jupiter, for instance), and possibly without the penalty of added structural weight, since intertial forces, and not just material tenacity, would contain the pressure-generated stresses.

The authors developed a zero-dimensional theory to predict specific impulse, and found it directly proportional to $[1 + (m_a/m)]^{1/2}$ where (m_a/m) is the ratio between the mass of ambient gas in nozzle and the mass of gas generated by explosion. This result has been found in the 1950's by Foa while exploring the potential of the pulsejet. The authors were also attracted by the relative compactness of the device, a feature that Oppenheim (1968) also had pointed out; however, he based it on the simple analysis of the detonation wave thickness, while in fact, the energy density of detonation waves is not larger, and actually may be smaller, than that of laminar flames at the same pressure. Thus, the actual size of the propulsion device will depend on considerations that take into account the wave thickness but are not necessarily determined by it.

The results of Back, Varsi and Kim investigations were applied to a simple conical nozzle which partially confined the explosive (1.5 g of Deta-sheet A (Fig. 4). For high cone angle (70 degrees) both experiments (with N_2 as inert gas filling the nozzle) and theory indicated a low Isp, of the order of 100 s. For 10 degree nozzle and N_2 or CO_2 as inert gas, a more elaborate conical flow theory showed good matching with the experiments, confirming the Isp growth with $[1 + (m_a/m]^{1/2}$ predicted by the idealized, zero-dimensional model (Fig. 5); in fact Isp reaches approximately 600 s for $(m_a/m) = 10^2$. No experiments were performed with inert gases beyond this value of (m_a/m). Four more tests were conducted where the nozzle was filled respectively with sand, epoxy microballoons, a mixture of microballoons and water, and water. The Isp, based on propellant mass only, was found to grow with density from 560 s to a maximum of 2100 s for the water case where $(m_a/m) = 209$. In the 1977 paper Kim, Varsi and Back computed the change of specific impulse with time, as the blast from the 10 degree nozzle sets in motion shock waves which are reflected at the exhaust end by the ambient gas (N_2 at 69 bars). Fig. 6 shows the specific impulse vs. time; in calculatng the specific impulse only the mass of the explosive has been included in the total mass expended, and its oscillations are due to the acceleration of colder gas entering periodically inside the nozzle as the waves move back and forth.

In the 1983 paper, Back, Dowler and Varsi presented new experimental data which were obtained with various types of nozzles: long cone, short cone, straight, and firing plug. With the long cone nozzle, there was a progressive increase in specific impulse with ambient pressure for the higher molecular weight gases, carbon dioxide and nitrogen, whereas for the lower molecular weight gases, helium and the simulated Jupiter atmosphere, the specific impulse decrease with increasing ambient pressure and the results were virtually independent of molecular weight of the ambient gas. These new data were analyzed using first principles, approximate predictions from blast wave theory, and numerical, two-dimensional calculations to acquire a basic understanding of the experimental trends and to predict specific impulses.

Although most of the previous work reviewed here deals with unsteady propulsive devices operating in an atmosphere, knowledge of the phenomena examined in these investigations opens the way to the understanding of the basic mechanisms involved in unsteady thrust generation and to the prediction of specific impulse.

V. Case 1: Constant Volume Explosion and Quasi-Steady Exhaust

In this case, the injected charge of propellant is burned instantaneously in a plenum and a uniform, very high combustion gas pressure is produced. The gas is then exhausted quasi-steadily through the nozzle while the plenum pressure remains uniform but decreases with time. The cycle is ended when the plenum pressure decreases to zero. The sketch of idealized rocket chamber is shown in Figure 7 where

V_c = volume of the combustion chamber

$m_{c,t}$ = mass of the combustion gas in the chamber at time t

$\dot{m}(t)$ = exhaust mass flow rate

$P_{c,t}$ = pressure of the combustion gas at time t

$P_{e,t}$ = nozzle exit pressure at time t

$T_{c,t}$ = temperature of the combustion gas at time t

$\rho_{c,t}$ = density of the combustion gas at time t

A_t = throat area of the nozzle

A_e = exit area of the nozzle

(A) Physical Assumptions and Simplifications

1) Ideal rocket assumptions are fulfilled (e.g., isentropic expansion etc.) except steady state assumption;

2) No wave motion;

3) No dilute gas in the chamber;

4) Burning time is short as compared with exhaust time;

5) Initial chamber pressure is high and then decreases with time;

6) The ratio of nozzle exit pressure to chamber pressure, $P_{e,t}/P_{c,t}$, is a constant.

Based on these assumptions the equations for ideal rocket can be applied to the present case except that terms such as pressure, temperature and residual mass of combustion gas are functions of time.

(B) Analysis

1) Governing Equations:

1. $\dot{m}(t) = A_t \, P_{c,t} \, \alpha / \sqrt{T_{c,t}}$

2. $m_{c,t} = m_{c,o} - \int \dot{m}(t) \, dt$

3. $\rho_{c,t} = m_{c,t}/V_c$

4. $T_{c,t} = \beta T_{c,o} \, (m_{c,t}/V_c)^{\gamma - 1}$

5. $P_{c,t} = (m_{c,t}/V_c) \, (R_{univ}/MW) \, T_{c,t}$

6. $V_c = m_{c,o} \, R \, T_{c,o}/P_{c,o}$

where

$$\alpha = \left\{ g \, \gamma \, [2/(\gamma + 1)]^{\frac{\gamma+1}{2(\gamma-1)}} \right\} / (g^{\gamma} R)^{1/2}$$

γ is defined as C_P/C_V

g is the gravitational acceleration constant

R = universal gas constant/molecular weight of combustion gas

$\beta = (\rho_{c,o})^{(1-\gamma)}$

2) Initial Equations

$P_{c,t} = P_{c,o}$

$T_{c,t} = T_{c,o}$

$m_{c,t} = m_{c,o}$

$\rho_{c,t} = \rho_{c,o}$

3) Input Values

γ = 1.2 and 1.3

molecular weight of combustion gas = 22.0 lb/mole

$m_{c,o}$ = 140 lb

$T_{c,o}$ = 6300 °R (3500 °K) and 5400 °R (3000 °K)

$$\Delta \equiv P_{e,t}/P_{c,t} = 0.01,\ 0.001$$

$$A_t = 0.0145 \text{ ft}^2$$

universal gas constant = 1544 ft-lb/mole $^{\circ}$R

From equations 1 through 6 one will obtain $P_{c,t}$ as a function of time. From A_t, $P_{c,t}$, P_e and γ, one would find the thrust as a function of time, i.e.,

$$F(t) = A_t\, P_{c,t}\, \Phi^{1/2} \left[1-(P_{e,t}/P_{c,t})^{\frac{\gamma-1}{\gamma}}\right]^{1/2}$$

where $\Phi = 2[2/(\gamma+1)]^{\frac{\gamma+1}{\gamma-1}}\ [\gamma^2/(\gamma-1)]$

The total impulse I_t and specific impulse I_s are defined as

$$I_t = \int_0^{\infty} F(t)\, dt$$

$$I_s = I_t/m_{c,o}$$

I_s can be obtained analytically as follows:

$$I_s = A_t \left(\sqrt{\Phi\ [1-\Delta^a]} + \Delta\varepsilon\right) (-DE_b)/(m_{c,o}\ b\ G)$$

where

$\varepsilon = A_e/A_t$, area ratio of nozzle

$D = \rho_{c,o}{}^{(1-\gamma)}\ T_{c,o}\ V_c{}^{(-\gamma)}\ R$

$E = m_{c,o}{}^{(1-\gamma)/2}$

$a = (\gamma-1)/\gamma$

$b = (1+\gamma)/(1-\gamma)$

$G = \left(\frac{(\gamma-1)}{2}\right) A_t\ R\ \alpha\ \ T_{c,o}{}^{1/2}\ \ \rho_{c,o}{}^{(1-\gamma)/2}\ V_c{}^{(-1-\gamma)/2}$

The values of I_s for two different values of γ, explosion energy (defined as the product of C_v and ($T_{c,o}$ - $T_{ambient}$)) and the ratio of $P_{e,t}/P_{c,t}$ are shown in Table 1. The corresponding values of specific impulse for steady state case are also presented in Table 1.

VI. Case 2: Constant Volume Explosion and Quasi-Steady Exhaust with Dilute Gas Added.

This case is the same as Case 1 except that the dilute gas is added just before explosion. The mixing process will affect the parameters such as the total mass, the temperature and the pressure of the gases in the combustion chamber. The new parameter introduced is the mass ratio W defined as

$$W = \frac{m_{d,o}}{m_{c,o}}$$

which is the ratio of the mass of dilute gas to that of the combustion gas at time equals zero.

(A) Physical Assumptions and Simplifications

Besides the assumptions and simplifications made in Case 1, we assume that the dilute gas and combustion gas are uniformly mixed, and they have the same thermodynamic properties. The molecular weights of the two gases are assumed to be the same. The mass ratio, W, is assumed to be a constant.

(B) Analysis

1) Symbols

$m_{c,t}$ = mass of the combustion gas at time t

$m_{d,t}$ = mass of the dilute gas at time t

W = $m_{d,t}/m_{c,t}$, mass ratio

$\dot{m}(t)$ = exhaust mass flow rate

$m_{mix,t} = m_{c,t} + m_{d,t}$

= total residual mass of the mixing gas at time t

V_c = volume of the combustion chamber

$_{mix,t}$ = density of the mixing gas at time t

$T_{mix,t}$ = temperature of the mixing gas in the combustion chamber at time t

$P_{mix,t}$ = pressure of the mixing gas in the combustion chamber at time t

$P_{c,t}$ = partial pressure of the combustion gas at time t

$P_{d,t}$ = partial pressure of the dilute gas at time t

$P_{e,t}$ = nozzle exit pressure at time t

$\Delta \equiv P_{e,t}/P_{c,t}$

A_e = exit area of the nozzle

A_t = area of the nozzle throat

$\varepsilon \equiv A_e/A_t$

2) Governing Equations

1. $\dot{m}(t) = A_t \, P_{mix,t} \, \alpha \, / (T_{mix,t})^{1/2}$

2. $m_{mix,t} = m_{mix,o} - \int \dot{m}(t) \, dt = m_{d,t} + m_{c,t}$

3. $\rho_{mix,t} = m_{mix,t}/V_c$

4. $T_{mix,t} = \beta T_{mix,o} \, (m_{mix,t}/V_C)^{\gamma-1}$

5. $P_{mix,t} = P_{d,t} + P_{c,t}$

6. $P_{d,t} = m_{d,t} \, R \, T_{mix,t}/V_c$

7. $P_{c,t} = m_{c,t} \, R \, T_{mix,t}/V_c$

8. $T_{mix,o} = (T_{c,o} + W \, T_{d,o})/(1+W)$

9. $V_c = m_{mix,o} \, R \, T_{mix,o}/P_{mix,o}$

where α and β were defined in Case 1.

3) Initial Equations

$m_{c,t} = m_{c,o}$

$m_{d,t} = m_{d,o}$

$m_{mix,t} = m_{mix,o}$

$\rho_{mix,t} = \rho_{mix,o}$

$T_{mix,t} = T_{mix,o}$

$P_{mix,t} = P_{mix,o}$

$P_{c,t} = P_{c,o}$

$P_{d,t} = P_{d,o}$

4) Input Values

$\gamma = 1.2$ and 1.3

molecular weight of gases = 22.0 lb/mole

$m_{c,o} = 140.$ lb

$\Delta = 0.01, 0.001$

$A_t = 0.0145$ ft^2

$W = 0, 1, 5, 10$

$T_{c,o} = 6300\ ^{o}R\ (3500\ ^{o}K)$ and $5400\ ^{o}R\ (3000\ ^{o}K)$

From equations 1 through 9 and following the similar derivation as in Case 1, one obtains $P_{mix,t}$ and the thrust F(t). Integrating F(t) one gets the total impulse I_t by

$$I_t = \int_0^{\infty} F(t)\, dt.$$

The specific impulse based on the initial mass of combustion gas is defined as

$$I_{sp}{}^{1} = I_t/m_{c,o}.$$

The specific impulse based on the total mass is defined as

$$I_{sp}{}^{2} = I_t/m_{mix,o}$$

The values of $I_{sp}{}^{1}$ and $I_{sp}{}^{2}$ for two different values of γ, explosion energy and pressure ratio, $P_{e,t}/P_{c,t}$, are shown in Tables 2 through 5. As the mass ratio (W) equals zero, Case 2 is identical to Case 1 as expected.

VII. Case 3. Straight-Duct Propulsor with Tail Pipe Intermittently Filled with Inert Material.

Assuming a straight duct propulsor divided by a divider which separates the combustion chamber and the tail pipe (see Figure 8), the tail pipe is filled with inert material intermittently. The charge of propellant inside the combustion chamber is burned instantaneously and a uniform, very high pressure is produced. The high combustion pressure pushes the divider moving to the exit of the tail pipe. The cycle is ended when the combustion pressure decreases to zero. The system of this case is shown in Figure 8, where

m_c = mass of the combustion gas in combustion chamber

$P_{c,t}$ = pressure of the combustion gas at time t

$T_{c,t}$ = temperature of the combustion gas at time t

V_c = volume of the combustion chamber

$U_{c,t}$ = the internal energy of the combustion gas at time t

V = velocity of the divider which separates the combustion chamber and the tailpipe

m_s = mass of the inert material in the tail pipe

$_s$ = density of the inert material

T_s = temperature of the inert material

$U_{s,t}$ = internal energy of the inert material at time t

W = mass ratio = m_s/m_c

A) Physical Assumptions and Simplifications

1) Divider is not permeable;

2) Velocity of the inert material is the same as that of the divider;

3) Velocity of the center of mass of the combustion gas is half of that of divider;

4) Isentropic expansion of the combustion gas in the combustion chamber;

5) Internal energy of the inert material is a constant.

(B) Analysis

1) Initial Equations

$T_{c,t} = T_{c,o}$

$P_{c,t} = P_{c,o}$

$U_{c,t} = U_{c,o}$

2) Governing Equations

By the principle of conservation of energy, the decrease of internal energy for the whole piston system is equal to the increase of kinetic energy for the whole piston system during the expansion process.

1. $$\Delta U = (m_c\, U_{c,o} + m_s\, U_s) - (m_c\, U_{c,t^*} + m_s\, U_s)$$
$$= m_c\, (U_{c,o} - U_{c,t^*})$$

2. $$\Delta E_k = (1/2)\, m_c\, (V/2)^2 + (1/2)\, m_s\, V^2$$

where t* is the time when the chamber pressure equals zero.

3) Input Values

$\gamma = 1.2$ and 1.3

molecular weight of combustion gas: 22.0 lb/mole

$m_{c,o} = 140.$ lb

$T_{c,o} = 6300\ ^oR$ ($3500\ ^oK$) and $5400\ ^oR$ ($3000\ ^oK$)

$W = 0, 1, 5, 10$

PR = final chamber pressure/initial chamber pressure = 0

$\rho_s = 0.0765$ lb/ft^3

Equating ΔU and ΔE_k, one obtains the velocity of the divider. The specific impulse based on the mass of combustion gas is

$$I_{sp}^1 = [(1/2)\, m_c\, V + m_s\, V]/m_c$$

The specific impulse based on the total mass is

$$I_{sp}^2 = [(1/2)\, m_c\, V + m_s\, V]/(m_c + m_s)$$

The results of I_{sp}^1 and I_{sp}^2 for the same values of γ and explosion energy as Case 1 and 2 are shown in Tables 6 through 9.

VIII. Conclusions

On the basis of the performance analyses reported herein, three general conclusions can be drawn:

(1) An intermittent rocket engine cycle may show a specific impulse gain over that of a steady-state cycle. An intermittent engine also possesses a practical advantage in that it requires no propellant feed pressurization system; the explosion inherent in the intermittent system provides the desired level of chamber pressure according to the mass injected at the start of the cycle.

(2) Addition of inert mass to supplement the driving energetic propellant charge can add substantially to the thrust from a given size engine and to the effective specific impulse (based on the propellant charge mass alone). This leaves open the question of whether there will indeed exist any so-called waste mass for disposal in this manner on a spacecraft. It does not matter, in principle, what form, solid, liquid, or gas, that the added inert mass has, but it must be possible to mix it effectively with the propellant gas or to bring about effective pressure contact between the two. If no added mass is available, then the conclusion (1) applies.

(3) An open question remains, whether a more complex analysis involving detailed step-by-step computation of explosive (or detonative) wave motions, with optimized geometric ducts, can indeed show a higher specific impulse than the values obtained with the simplifying assumptions used here. Unfortunately, the quasi-steady types of analysis used in this study cannot answer that question, although we tend to believe that the performance gain, if any, will not be large. The advantages of such detonative propulsion may not lie in much gain in specific impulse but in practical design advantages resulting from a less cumbersome feed system. (A steady-state rocket feed system is penalized also by using some of the combustion energy, for pressurization, to be sure, but this penalty is usually negligible unless very high rocket pressures are contemplated, well above optimum values.)

IX. Recommendations

A principal open question left unanswered by this brief study is whether a duct of optimized configuration operating on a truly non-steady combustion-and-outflow cycle (e.g., a true deflagration or detonation process) can show any higher specific impulse than an equivalent steady-state process. The assumption made herein -- necessary to stay within the confines of a very brief study -- that the cycle is quasi-steady may have resulted in a underestimate of the performance. To investigate this question analytically, a procedure involving step-by-step computer calculations, with various assumptions as to the process and with various possible duct configurations, will have to be made. In the absence of such analysis, the only evidence that is at hand is the experimental finding that a straight cylindrical duct, filled with an explosive mixture of stoichiometric hydrogen-oxygen gas and then detonated, delivered a specific impulse of only 50% of that of an equivalent steady-state rocket with the same propellants (Cf. Section IV, literature review section). But this limited test result may not be representative of the full potential of such non-steady rocket propulsion, for various practical reasons. Thus, it is our recommendation that, if a continuation of the search for improved performance is to be carried out, it should be based on a comprehensive non-steady analysis. One such approach is discussed in Appendix A.

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TABLE 1. Specific Impulse for Steady State and Quasi-State Calculation (in sec.).

	$\gamma=1.2$				$\gamma=1.3$			
Q	2599.67		2193.48		1733.1		1462.31	
mode PR	QS	SS	QS	SS	QS	SS	QS	SS
0.01	289.7	278.2	268.2	255.5	257.5	248.4	238.4	228.2
0.001	317.0	304.4	293.5	279.6	275.7	265.9	255.3	244.3

Q : Explosion Energy (Btu/Lb)

QS : Quasi-steady state

SS : steady state

PR : Pressure ratio = $P_{e,t} / P_{c,t}$

(Exit pressure / Chamber pressure)

TABLE 2. Specific Impulse for Quasi-Steady Exhaust with Dilute Gas Added.

$$m_d/m_c = 0.$$

	$\gamma=1.2$				$\gamma=1.3$			
Q	2599.67		2193.48		1733.11		1462.31	
PR	I_{sp}^{1}	I_{sp}^{2}	I_{sp}^{1}	I_{sp}^{2}	I_{sp}^{1}	I_{sp}^{2}	I_{sp}^{1}	I_{sp}^{2}
0.01	289.7	289.7	268.2	268.2	257.5	257.5	238.4	238.4
0.001	317.0	317.0	293.5	293.5	275.7	275.7	255.3	255.3

Q : explosion energy (Btu/Lb)

PR : exit pressure / chamber pressure = $P_{e,t}/P_{c,t}$

I_{sp}^{1} : specific impulse based on the mass of combustion gas only

I_{sp}^{2} : specific impulse based on the total mass of gas mixture

TABLE 3. Specific Impulse for Quasi-Steady Exhaust with Dilute Gas Added.

$$m_d/m_c = 1.$$

	$\gamma=1.2$				$\gamma=1.3$			
Q	2599.67		2193.48		1733.11		1462.31	
PR	I_{sp}^1	I_{sp}^2	I_{sp}^1	I_{sp}^2	I_{sp}^1	I_{sp}^2	I_{sp}^1	I_{sp}^2
0.01	426.9	213.5	397.8	198.9	379.5	189.8	353.7	176.8
0.001	467.1	233.6	435.3	217.7	406.3	203.2	378.7	189.3

Q : explosion energy (Btu/Lb)

PR : exit pressure / chamber pressure = $P_{e,t}/P_{c,t}$

I_{sp}^1 : specific impulse based on the mass of combustion gas only

I_{sp}^2 : specific impulse based on the total mass of gas mixture

TABLE 4. Specific Impulse for Quasi-Steady Exhaust with Dilute Gas Added.

$$m_d / m_c = 5.$$

	$\gamma = 1.2$				$\gamma = 1.3$			
Q	2599.67		2193.48		1733.11		1462.31	
PR	I_{sp}^{1}	I_{sp}^{2}	I_{sp}^{1}	I_{sp}^{2}	I_{sp}^{1}	I_{sp}^{2}	I_{sp}^{1}	I_{sp}^{2}
0.01	848.2	141.4	804.7	134.1	754.0	125.7	715.3	119.2
0.001	928.1	154.7	880.5	146.7	807.3	134.6	765.9	127.6

Q : explosion energy (Btu/Lb)

PR : exit pressure / chamber pressure = $P_{e,t} / P_{c,t}$

I_{sp}^{1} : specific impulse based on the mass of combustion gas only

I_{sp}^{2} : specific impulse based on the total mass of gas mixture

TABLE 5. Specific Impulse for Quasi-Steady Exhaust with Dilute Gas Added.

$$m_d / m_c = 10.$$

	$\gamma = 1.2$				$\gamma = 1.3$			
Q	2599.67		2193.48		1733.11		1462.31	
PR	I^1_{sp}	I^2_{sp}	I^1_{sp}	I^2_{sp}	I^1_{sp}	I^2_{sp}	I^1_{sp}	I^2_{sp}
0.01	1309.	119.0	1258.	114.4	1164.	105.8	1118.	101.7
0.001	1433.	130.3	1377.	125.1	1246.	113.3	1197.	108.9

Q : explosion energy (Btu/Lb)

PR : exit pressure / chamber pressure = $P_{e,t} / P_{c,t}$

I^1_{sp} : specific impulse based on the mass of combustion gas only

I^2_{sp} : specific impulse based on the total mass of gas mixture

TABLE 6. Specific Impulse for Straight-Duct Propulsor with Slug Filled in Tail Pipe.

$$m_s/m_c = 0$$

	$\gamma = 1.2$				$\gamma = 1.3$			
Q	2599.67		2193.48		1733.11		1462.31	
PR	I_{sp}^{1}	I_{sp}^{2}	I_{sp}^{1}	I_{sp}^{2}	I_{sp}^{1}	I_{sp}^{2}	I_{sp}^{1}	I_{sp}^{2}
0.	370.7	370.7	343.2	343.2	302.7	302.7	280.2	280.2
RATIO	1	1	1	1	1	1	1	1

Q : explosion energy (Btu/Lb)

PR : ratio of the final to initial Pressure of the combustion gas in combustion chamber

I_{sp}^{1} : specific impulse based on the mass of combustion gas only

I_{sp}^{2} : specific impulse based on the total mass (combustion gas + slug)

RATIO : ratio of the present case to the corresponding case without

TABLE 7. Specific Impulse for Straight-Duct Propulsor with Slug Filled in Tail Pipe.

$$m_s/m_c = 1$$

	$\gamma = 1.2$				$\gamma = 1.3$			
Q	2599.67		2193.48		1733.11		1462.31	
PR	I_{sp}^1	I_{sp}^2	I_{sp}^1	I_{sp}^2	I_{sp}^1	I_{sp}^2	I_{sp}^1	I_{sp}^2
0.	497.3	248.7	460.4	230.2	406.1	203.0	375.9	188.0
RATIO	1.34	0.67	1.34	0.67	1.34	0.67	1.34	0.67

Q : explosion energy (Btu/Lb)

PR : ratio of the final to initial pressure of the combustion gas in combustion chamber

I_{sp}^1 : specific impulse based on the mass of combustion gas only

I_{sp}^2 : specific impulse based on the total mass (combustion gas + slug)

RATIO : ratio of the present case to the corresponding case without

TABLE 8. Specific Impulse for Straight-Duct Propulsor with Slug Filled in Tail Pipe.

$$m_s/m_c = 5$$

	$\gamma=1.2$				$\gamma=1.3$			
Q	2599.67		2193.48		1733.11		1462.31	
PR	I_{sp}^{1}	I_{sp}^{2}	I_{sp}^{1}	I_{sp}^{2}	I_{sp}^{1}	I_{sp}^{2}	I_{sp}^{1}	I_{sp}^{2}
0.	889.8	148.3	823.8	137.3	726.5	121.1	672.6	112.1
RATIO	2.40	0.40	2.40	0.40	2.40	0.40	2.40	0.40

Q : explosion energy (Btu/Lb)

PR : ratio of the final to initial pressure of the combustion gas in combustion chamber

I_{sp}^{1} : specific impulse based on the mass of combustion gas only

I_{sp}^{2} : specific impulse based on the total mass (combustion gas + slug)

RATIO : ratio of the present case to the corresponding case without

TABLE 9. Specific Impulse for Straight-Duct Propulsor with Slug Filled in Tail Pipe.

$$m_A / m_c = 10$$

	$\gamma = 1.2$				$\gamma = 1.3$			
Q	2599.67		2193.48		1733.11		1462.31	
PR	I_{sp}^1	I_{sp}^2	I_{sp}^1	I_{sp}^2	I_{sp}^1	I_{sp}^2	I_{sp}^1	I_{sp}^2
0.	1215.7	110.5	1125.5	102.3	992.6	90.2	919.0	83.5
RATIO	3.28	0.30	3.28	0.30	3.28	0.30	3.28	0.30

Q : explosion energy (Btu/Lb)

PR : ratio of the final to initial pressure of the combustion gas in combustion chamber

I_{sp}^1 : specific impulse based on the mass of combustion gas only

I_{sp}^2 : specific impulse based on the total mass (combustion gas + slug)

RATIO : ratio of the present case to the corresponding case without

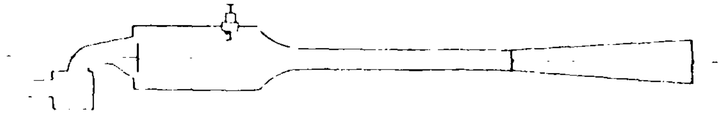


FIG. 1. THE REACTEUR-PULSATEUR OF MARCONNET (1910)

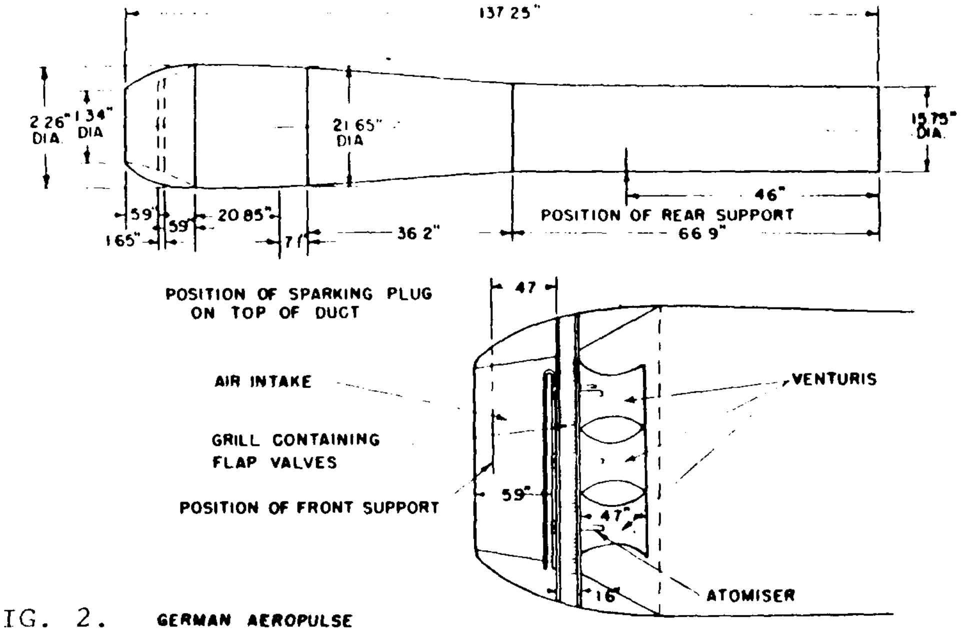

137 25"
2 26" DIA.
1 34" DIA
21 65" DIA
15 75" DIA.
46"
POSITION OF REAR SUPPORT
66 9"
59"
59"
1 65"
20 85"
7 1"
36 2"
47
59"
47"
16"
POSITION OF SPARKING PLUG ON TOP OF DUCT
AIR INTAKE
GRILL CONTAINING FLAP VALVES
POSITION OF FRONT SUPPORT
VENTURIS
ATOMISER

FIG. 2. **GERMAN AEROPULSE**
GENERAL ARRANGEMENT OF DUCT

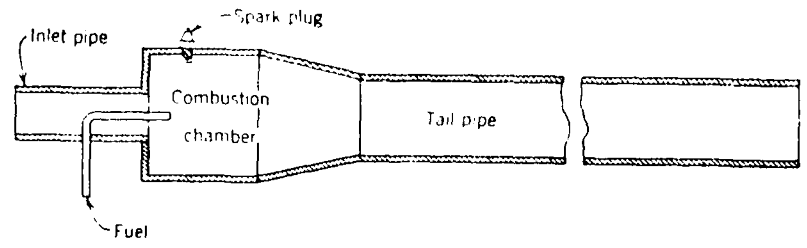

Inlet pipe
Spark plug
Combustion chamber
Tail pipe
Fuel

FIG. 3. DUNBAR-HUSSEY-SCHUBERT PULSEJET (Foa, 1960)

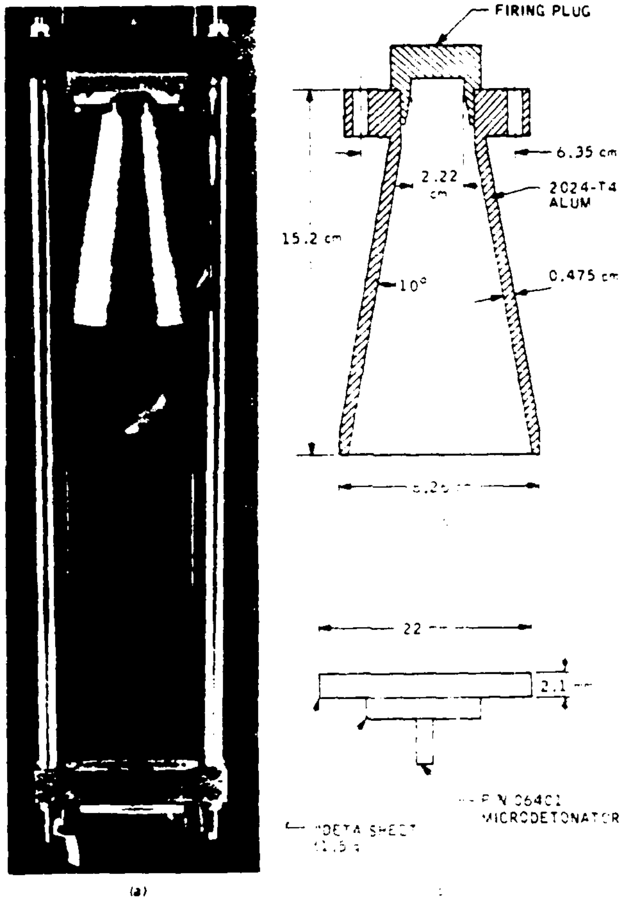

FIRING PLUG
6.35 cm
2.22 cm
2024-T4 ALUM
15.2 cm
10°
0.475 cm
5.25 cm
22
2.1
P/N 06401 MICRODETONATOR
DETA SHEET 12.5 g
(a)

FIG. 4. NOZZLE FOR DETONATION THRUSTER (from Varsi and Back, 1973)

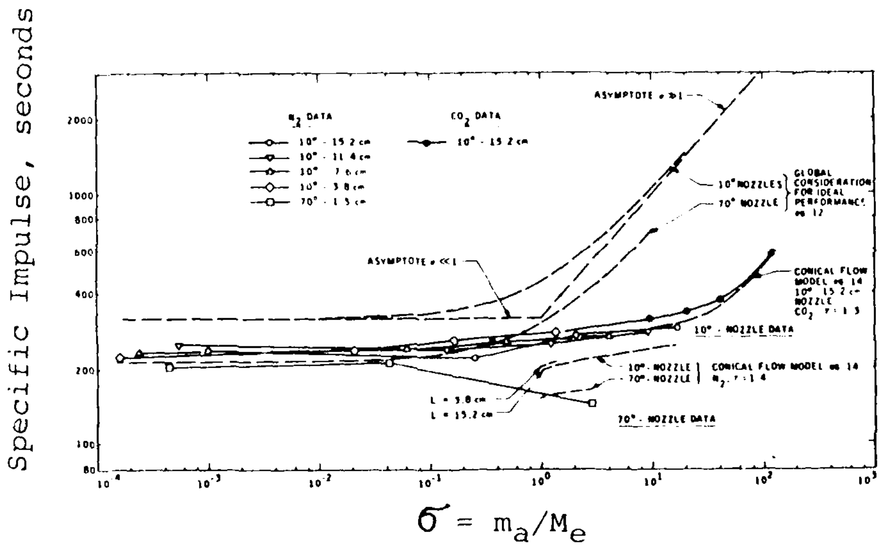


FIG. 5. MEASURED AND PREDICTED PROPULSION PERFORMANCE FOR DETONATION THRUSTER (Back and Varsi, 1974)

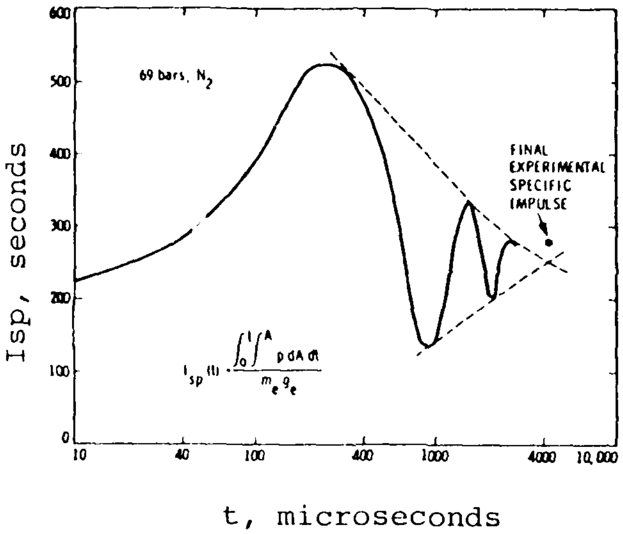


FIG. 6. TRANSIENT SPECIFIC IMPULSE FOR A DETONATION THRUSTER (Kim et al, 1977)

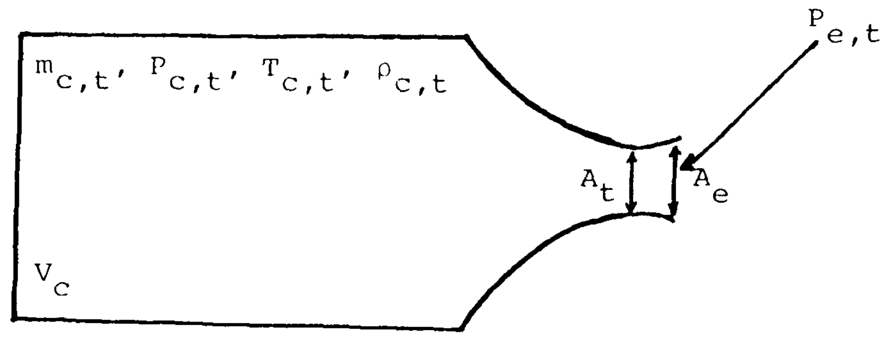

$P_{e,t}$
$m_{c,t}$, $P_{c,t}$, $T_{c,t}$, $\rho_{c,t}$
A_t
A_e
V_c

FIG. 7. SKETCH OF THE IDEALIZED ROCKET IN CASE 1.

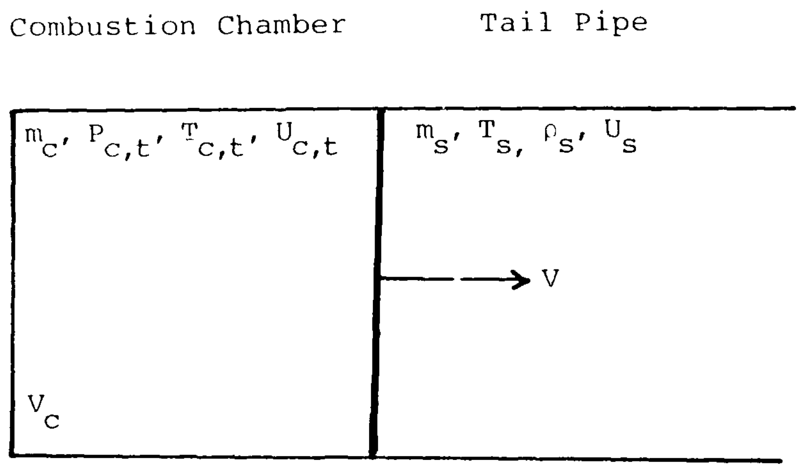

Combustion Chamber
Tail Pipe
m_c, $P_{c,t}$, $T_{c,t}$, $U_{c,t}$
m_s, T_s, ρ_s, U_s
V
V_c

FIG. 8. SKETCH OF THE STRAIGHT-DUCT IN CASE 3.

APPENDIX A

Detonative Propulsion for Rapid Space-Platform Orbit Change: A Possible Approach for Follow-on Efforts·

1. Introduction

One critical aspect of space platform maneuverability is the performance of rapid orbital changes. These would typically require tens or hundreds of tons thrust, for short periods of time, of the order of 0.1 seconds. Whereas electrical, ion and MHD thrusters afford great mass efficiency (very high specific impulse, Isp), the thrusts attainable at present are still very low. In looking for viable technological solutions applicable in the immediate future (say, within 10 years), chemical propulsion holds a clear advantage over other propulsion concepts.

This study is focused on the concept of detonative or explosive propulsion. In this mode, a large amount of chemical energy can be imparted to gaseous products over a very short period of time, leading to high pressures and eventually, very high gas velocities. Unfortunately, there are several limitations despite the high thrusts attainable. To mention a few: very short durations (which would require intermittent, periodic operation), very low overall mass-efficiency (low mean Isp is characteristic), and the inconvenience of very high local stress in the blast-reflector unit. These serve to motivate the present feasibility study.

The objectives to this analysis are as follows: (1) identify the major physical interactions, the important physical parameters, and their effects on performance; (2) define idealized reference impulse-motor configurations, to provide benchmark cases for evaluation of performance and overall energy conversion efficiency of more realistic detonative/explosive propulsion units; and (3) outline a method to generate optimal thruster geometry (or blast reflector unit design), to optimize thrust while energy losses are kept at minimum; typical constraints: the available chemical energy, C-J detonation speeds, the maximal allowable stress in the reflector.

The approach adopted is as follows. Incorporate, within an analytical model, the thermodynamic and fluid-dynamic processes relevant to strong explosions in vacuum, shock reflection and elastic reflector surface admittance. Analytical solutions (not necessarily closed-form) are sought for the pressure and velocity fields, for particular simplifying assumptions. These explicit expressions are then used to generate total and specific impulse and thrust, from the relevant integral expressions, which, in turn, are utilized in the thrust acceptor geometry optimization study.

The following results are anticipated: (1) point out the physical parameters in control of performance and efficiency, and their effect. This would be accomplished by approximate analytical calculations of total impulse, specific impulse, net thrust vs. time and the overall energy conversion efficiency; (2) through optimization analysis, show whether the chemical detonative propulsion concept is viable, and under what parameter range (or physical conditions) could performance be maximized; (3) outline developmental designs, based on the optimal features derived in the present study.

2. A Hypothetical Rapid Maneuver

The following hypothetical rapid orbit change is considered as follows. An unmanned space platform is required to momentarily translate a distance of $\Delta X = 1$ m out of orbit, within $\Delta t = 0.1$ sec. The associated mean velocity change and linear acceleration are, respectively

$$\Delta V = \Delta X / \Delta t = 10 \text{ m/sec}$$

$$a_c \doteq \Delta V / \Delta t = 100 \text{ m/sec}^2$$

For a spacecraft mass of $m_s = 1000$ kg, the net required thrust for execution of this maneuver would be

$$\overline{F}_R \doteq m_s \, a_c = 10^5 \text{ Newt} \qquad (10^4 \text{ kgf})$$

Thrusts of the order of 10 tons force can be achieved at present only by chemical propulsion systems. The relatively short duration ($\Delta t \doteq 0.1$ sec) further indicates that impulsive (detonative or explosive) operation could be desirable, perhaps in a repeating mode.

The plausibility of a detonative chemical propulsion system for execution of the foregoing typical maneuver can be demonstrated by the following simplified calculations.

3. Explosive Charge with Flat Disk Reflector

Consider a propulsion unit with a circular reflector of radius $L = 0.5$ m. A small solid charge with specific explosion energy of $e^o = 1000$ cal/g, is aligned with the reflector center. Following detonation, a spherical material wavefront is formed, behind which the gas is expanding and accelerating; it rapidly attains a constant speed, approximated as

$$u_f \sim \sqrt{2E^o/M_D} = \sqrt{2e^o} = 2{,}900 \text{ m/sec}$$

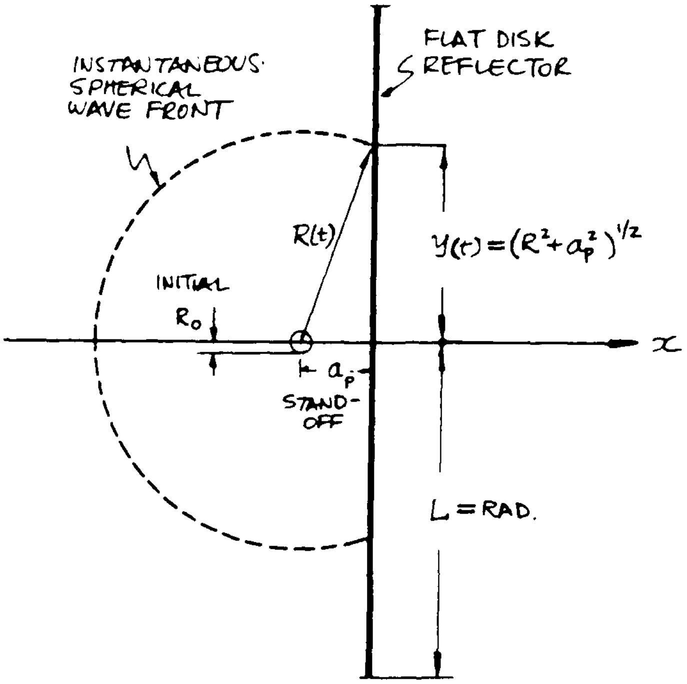

INSTANTANEOUS SPHERICAL WAVE FRONT
FLAT DISK REFLECTOR
R(t)
$y(t)=(R^2+a_p^2)^{1/2}$
INITIAL R_o
x
a_p
STAND-OFF
L = RAD.

Fig. 1 Spherical blast wave produced by detonating small charge at a standoff distance from a flat plate reflector; reflected shock wave not shown. Total impulse and thrust vs. time are calculated by using the mean recovery pressure and instantaneous exposed area, $P_m(t) \quad y(t)^2$.

where E^o and M_p denote the total explosion energy and the charge mass, respectively. The material front will cover the radial distance of the reflector within

$$t_f \sim L/u_f \doteq 0.17 \text{ msec}$$

The following simplifying assumptions are now made: (1) the pressure $p_m(t)$ within the expanding gas sphere is nearly uniform, this pressure, $p_m(t)$ also serves as an approximation for the instantaneous pressure at the reflector surface, behind the reflected shock wave, (3) the expansion process is nonisentropic in general, and (4) the material wave front is moving at a nearly constant speed, $u_f \simeq (2E^o/M_p)^{0.5}$, as calculated earlier. For this system, then

$$p_m(t) \doteq p_c \Big/ \left(\frac{R(t)}{R_o} \right)^{3\gamma}$$

as shown by Zeldovich and Raizer (1966), pp. 104-106.R(t) = the instantaneous material front radius, and R_o = initial charge radius. Assumption (4) yields

$$R(t)/R_o = (u_f/R_o)t + 1$$

The initial pressure at the center, p_c, is calculated from the energy at the instant of fully-detonated charge, assuming calorically-perfect gas:

$$p_c = \frac{\gamma - 1}{\frac{4}{3}\pi R_o^3} E^o \doteq 6 \times 10^8 \text{ Newt/m}^2 = 600. \text{ MPa}$$

The numbers were generated by assuming $\gamma = 1.2$, $m_p = 0.01$ kg, $E^o = M_p e^o = 4.184 \times 10^4$ Joule, and $R_o = 0.015$ m.

It should be emphasized that $p_m(t)$ herein is not a static pressure. The proposed expression is an attempt to evaluate the recovery pressure at the wall - without the complexity of computing the three dimensional, nonsteady flow field behind the reflected shock.

Consider now a rigid flat disk reflector with the explosive charge at a standoff distance of a_p from he disk center. As the explosion proceeds, a larger and larger area A(t) is exposed to

$p_m(t)$, which is uniform but decreases rapidly as the 3γ -power of the cloud radius. The total impulse is,

$$I_o = \int_{t_o}^{t_1} F(t)dt = \int_{t_o}^{t_1} \left(p_m(t) - p_e\right) A(t)\,dt$$

$$A(t) = \pi\left[R(t)^2 - a_p^2\right]$$

and where t_o and t_1 denote the times, respectively, when the expanding gas cloud first reaches the reflector plate, $R(t_o) = a_p$, and when it expands beyond it, $R(t_1) = (L^2 + a_p^2)^{0.5}$. The external counter-pressure is $p_e = 0$. Transform to R/R_o as independent variable and use of the definition of $p_m(t)$ yields:

$$I_o = \frac{p_c \pi R_o^3}{u_f} \int_{Z_o}^{Z_1} Z^{-3\gamma}(Z^2 - Z_o^2)\,dZ$$

$$= p_c \frac{\pi R_o^3}{u_f} \left\{ \frac{Z^{3(1-\gamma)}}{3(1-\gamma)} - \frac{Z_o^2 Z^{(1-3\gamma)}}{1-3\gamma} \right|_{Z_o}^{Z_1}$$

where $Z \equiv R/R_o$, $dt \equiv (R_o/u_f)dZ$, while $Z_o \equiv a_p/R_o$ and $Z_1 \equiv L/R_o$. Assuming that $L/R_o >> 1$ while $Z_o \sim 0(1)$, the following first approximation is obtained:

$$I_o \simeq p_c \frac{\pi R_o^3}{u_f} \frac{2/3}{(\gamma-1)(3\gamma-1)} Z_o^{3(1-\gamma)}$$

Using now the foregoing definitions of p_c and u_f in terms of the available specific explosion energy, one may further write

$$I_o \simeq \frac{M_p\sqrt{2e^o}}{4(3\gamma-1)} (R_o/a_p)^{3(\gamma-1)}$$

$$I_{sp} \doteq I_o/(M_p g) \simeq \frac{\sqrt{2e^o}/g}{4(3\gamma-1)} (R_o/a_p)^{3(\gamma-1)}$$

$$\bar{F}_m = I_o/t_f = I_o u_f/L \simeq \frac{M_p e^o/L}{2(3\gamma-1)} (R_o/a_p)^{3(\gamma-1)}$$

Assuming now that $Z_o = a_p/R_o \simeq 1$, the previously specified reference values for e^o, M_p, γ and L yield: $I_o \simeq 2.8$ Newt-sec, Isp $\simeq$ 28.5 sec, and $F_m \simeq 1.6 \times 10^4$ Newt.

The important results of the present approximate analysis are the explicit trends obtained for the total impulse I_o, the specific impulse Isp, and the mean (over a pulse cycle) thrust, $\overline{F}_m$ all of which are expressed in terms of the basic physical parameters of the propulsion system considered. Regardless of its accuracy in actual simulation of a flat disk response to a spherical detonation in vacuum, the analysis points out the basic deficiency of the system (low efficiency as shown by low Isp), as well as its relative merit (very high thrust, although only for a short period of time). Further, it can immediately result in improved correlations of experimental observations, e.g., in direct impulse measurements. In comparison, the analysis by Baker (1982 AIAA Propulsion Meeting) contains "correlation" for I_o and Isp which is completely independent of γ and R_o.

Perhaps most important is the observation that, despite its deficiencies, this simple device is capable of thrust levels only slightly smaller than that required for a rapid orbit change maneuver. This indicates that improved efficiency (through optimal reflector geometry, product gas properties, etc.) and repeatable operation may enable practical use of such simple systems.

4. Idealized Reference-Motors

In preparation for systematic analysis of chemical-impulsive propulsion systems, it is necessary to establish idealized reference-cases in terms of efficiency and performance. These will serve as benchmarks, against which the impulse, specific impulse, mean thrust, etc., of various configurations, will be tested.

Two of these cases comprise of conventional rocket motors, with quasi-steady nozzles. The following assumptions are made: (1) equilibration (timewise) within the nozzle is much more rapid than any other variation in ths system, (2) nozzle throat is choked at all times, (3) the expansion process is adiabatic and shockless throughout.

The difference between the two reference cases is the chamber pressure, p_c: in Case A, $p_c(t)$ = const, while in Case B, $p_c(t)$ is variable in a particular manner.

The quasi steady nozzle assumption implies no mass accumulation within the nozzle, so that at any point along its axis,

$$\rho u A \big|_x = (\rho u A)_t$$

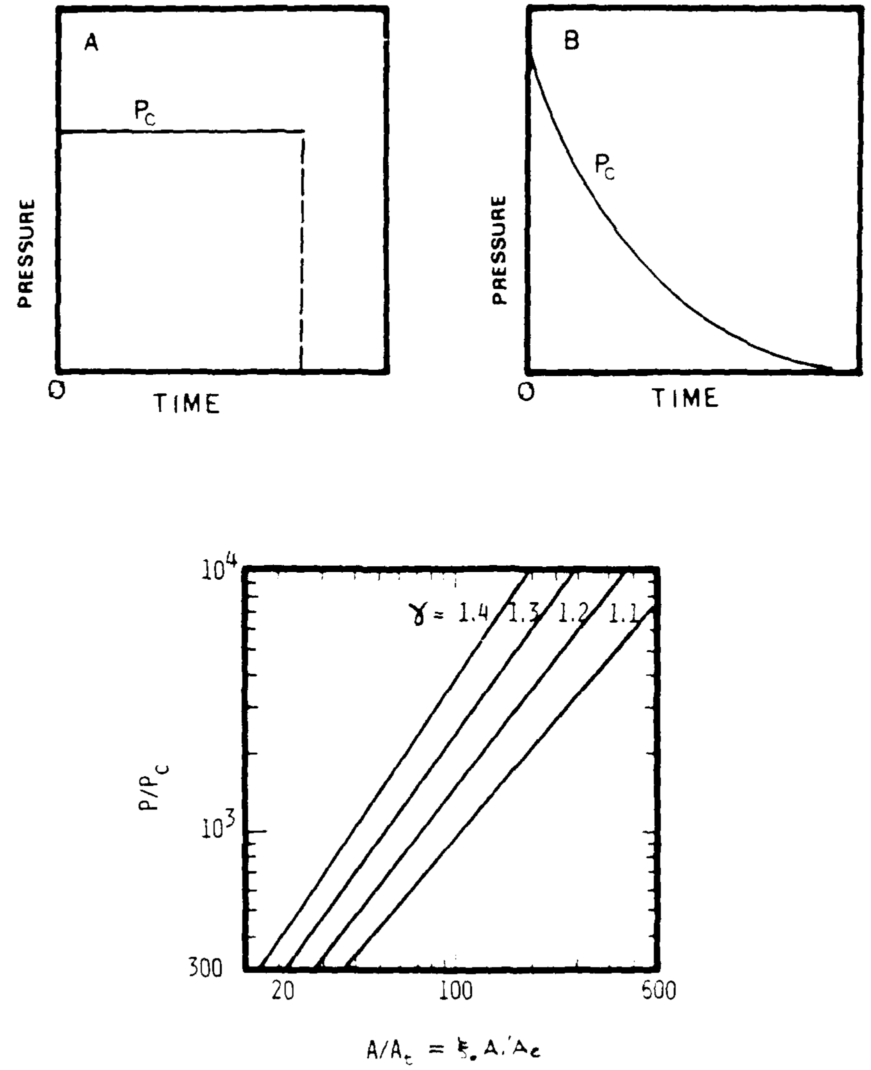


Fig. 2 The two reference-motor configurations (A,B) shown conceptually, through the chamber pressure histories. Both use a quasi-steady isentropic nozzle, for which the functional relationship between local $P/P_C(t)$ and A/At is shown, for various values of the ratio of specific heats, γ. These functional relations are used in the thrust and total impulse integrals, for At/Ae << 1.

where $(\ \ )_t$ denotes the choked throat position. For isentropic expansion, this leads to

$$A/A_t = C_G(\gamma)\left[1-(p/p_c)^{\frac{\gamma-1}{\gamma}}\right]^{-1/2}(p/p_c)^{-1/\gamma}$$

$$\equiv g_1(\gamma; p/p_c)$$

where

$$C_G(\gamma) \equiv \left(\frac{\gamma-1}{2}\right)^{1/2}\left(\frac{2}{\gamma+1}\right)^{\frac{1+\gamma}{2(\gamma-1)}}$$

thus,

$$A/A_e = \xi_o\, g_1(\gamma; p/p_c)$$

where ξ_o = At/Ae, the total expansion ratio.

Using f_c, the inverse function to g_1, one may invert the last expression to obtain

$$p/p_c = f_c(\gamma, \xi_o; A/A_e)$$

assuming the inverse function f_c exists and is Lipshitz-continuous in the region

$$0 < \xi \equiv A/A_e < 1$$

In practice, obtaining $p/p_c = f_c\ (A/Ae)$ explicitly is quite difficult, for obvious reasons; it may, however, be readily obtained numerically.

The instantaneous thrust and the total impulse may therefore be defined for all cases employing such quasi-steady nozzles:

$$F(t) = \int_{A_t}^{A_e} p\,dA = A_e\int_{\xi_o}^{1} p_c(t)\,f_c(\gamma, \xi_o; \xi)\,d\xi$$

$$I_o = \int_{t_o}^{t_f} F(t)\,dt = A_e\int_{t_o}^{t_f} p_c(t)\,dt\int_{\xi_o}^{1} f_c(\gamma, \xi_o; \xi)\,d\xi\ .$$

Note that $Ae = \pi L^2$ where L, the exit radius, is a fundamental dimensional parameter of the system. It should be emphasized that the quasi-steady nozzle expansion assumption cannot be justified for very high rates of gas generation in the chamber, which are relevant to the present study.

a. Case A. $P_c(t)$ = CONST

This configuration may be obtained, ideally, if the rates of gas generation in the chamber and exhaust through the nozzle were equal at all times (just like the steady state rocket motor operation). Thus,

$$-(\rho u A)_t = \dot{M}_p \qquad kg/sec$$

Suppose now that the linear gasification rate (or burning velocity) of the propellant is v_b (m/sec), its characteristic dimension is R_o(m), and its total mass, M_p (kg). Then, the total burn time and mean mass burning rate are, respectively,

$$t_f = R_o/v_b$$

$$\dot{M}_p = M_p/t_f = -M_p v_b/R_o$$

The prevailing temperature in the chamber is (assuming zero gas velocity) the isobaric adiabatic flame temperature.

$$T_p \simeq q^o/\bar{C}_p + T^o = \text{const.}$$

Where q^o (J/kg) is the specific heat of explosion of the propellant, at the reference temperature T^o (K), and $\bar{C}_p$ (J/kg-K) is the mean isobaric specific heat of the product gas mixture.

The left hand side of the overall mass balance, for adiabatic, quasi-steady flow,

$$(\rho u A)_t = \Gamma(\gamma)\,\frac{\xi_o A_e}{\sqrt{RT_p}}\,p_c = \Gamma\left(\frac{2\gamma}{\gamma-1}\right)^{1/2}\frac{\xi_o A_e}{\sqrt{2q^o}}\,p_c$$

$$\Gamma(\gamma) \equiv \sqrt{\gamma}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} ; \quad T_p \simeq q^o/\bar{C}_p \,.$$

Therefore, $p_c(t)$ can be computed from combination of the last expression with M_p:

$$p_c(t) = \frac{M_p v_b \sqrt{2q^o}/(R_o \xi_o A_e)}{\left(\frac{2\gamma}{\gamma-1}\right)^{1/2} \Gamma(\gamma)} = const$$

This expression for p_c (N/m^2) can then be used for thrust and total impulse calculations.

b. Case B. Variable $p_c(t)$:

This configuration may be obtained ideally by assuming that the propellant charge is entirely gasified in one instant, by constant-volume explosion. Following this event, the gas expands gradually as the chamber empties through the choked nozzle. In this instance, the overall chamber mass balance yields

$$\frac{dM_c}{dt} = V_c \frac{d\rho_c}{dt} = -(\rho u A)_t$$

where V_c = the chamber volume. For adiabatic process in the chamber, and assuming ideal gas, the right hand side is written as

$$(\rho u A)_t = \Gamma \xi_o A_e \, p_c / \sqrt{R T_c} = \Gamma \xi_o A_e \sqrt{p_c \rho_c}$$

$$= \Gamma \xi_o A_e \sqrt{B_o} \cdot \rho_c^{\frac{\gamma+1}{2}} \equiv K_o \rho_c^{\frac{\gamma+1}{2}}$$

where $p_c = B_o \rho_c^{\gamma}$ has been used.

The differential equation is therefore,

$$\frac{d\rho_c}{dt} = -K_o \rho_c^{\frac{\gamma+1}{2}}, \quad \rho_c(0) = \rho_o$$

The solution is:

$$\rho_c^{-\frac{\gamma-1}{2}} \Big|_{\rho_o}^{\rho(t)} = \frac{\gamma-1}{2} K_o t$$

After some manipulation:

$$p_c(t) = p_0/(1+K_1 t)^{\frac{2\gamma}{\gamma-1}}$$

$$K_1 = \frac{\gamma-1}{2} K_0 \rho_0^{\frac{\gamma-1}{2}}$$

where, for $\gamma = 1.2$, the exponent $2\gamma/(\gamma-1) = 12$, and

$$K_1 \equiv \left[\frac{\gamma-1}{2}\Gamma \xi_0 A_e/V_c\right]\sqrt{RT_0}$$

since

$$\rho_0^{\frac{\gamma-1}{2}}\sqrt{\beta_0} = (p_0/\rho_0)^{1/2} = (RT_0)^{1/2} .$$

Note that $V_c \sim R_o^3$ can be used again, with R_o being a relevant chamber dimension. Furthermore, the initial temperature, T_o is taken as the isochoric/adiabatic flame temperature: $T_o = T_v \sim q^o/\bar{C}_v$ where $\bar{C}_v$ is the isochoric specific heat of the gas mixture, and hence

$$\sqrt{RT_0} = \left(\frac{\gamma-1}{2}\right)^{1/2}\sqrt{2q^0}$$

so that finally we may write:

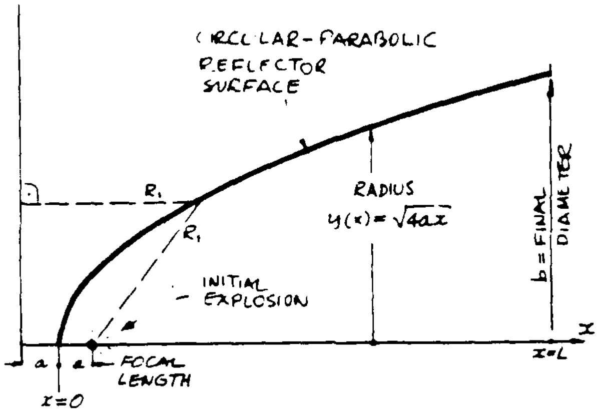
$$p_c(t) = p_0/[1+K_1 t]^{\frac{2\gamma}{\gamma-1}}$$

$$K_1 \equiv \left(\frac{\gamma-1}{2}\right)^{3/2}\Gamma(\gamma)\xi_0 A_e\sqrt{2q^0}/R_0^3$$

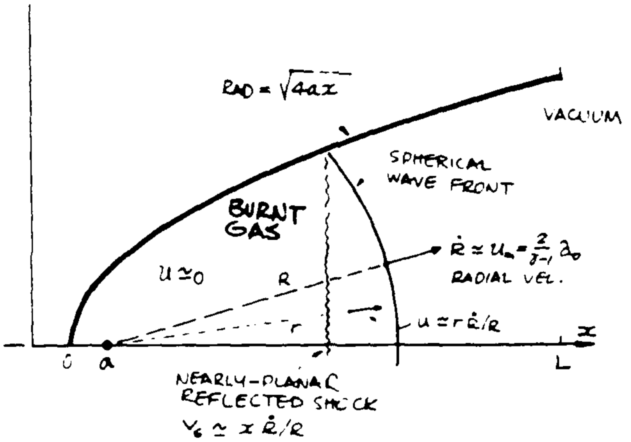
5. Nonsteady, Nonisentropic Expansion in Nozzle

Cases A and B described earlier have an obvious common deficiency: the quasi-steady nozzle assumption is indefensible whenever the exhaust processes are sufficiently fast.

With the aid of some (not too restrictive) simplifying assumptions, exact analytical solutions can be generated for the nonsteady case. The process under consideration is the rapid expansion of a gas cloud in vacuum, as restricted by an


CIRCULAR-PARABOLIC
REFLECTOR
SURFACE
R_1
R_1
RADIUS
$y(x) = \sqrt{4ax}$
INITIAL
EXPLOSION
b = FINAL
DIAMETER
x
$x=L$
a
a
FOCAL
LENGTH
$x=0$

A. GEOMETRY AND CHARGE


$RAD = \sqrt{4ax}$
VACUUM
SPHERICAL
WAVE FRONT
BURNT
GAS
$u \simeq 0$
R
$\dot{R} \simeq u_m = \frac{2}{\gamma-1} a_0$
RADIAL VEL.
r
$u = r\dot{R}/R$
x
0
a
L
NEARLY-PLANAR
REFLECTED SHOCK
$V_s \simeq x\dot{R}/R$

B. PHYSICAL PROCESS: STRONG EXPLOSION IN VACUUM

Fig. 3 The geometry, and physical processes, corresponding to a paraboloid reflector geometry are shown. Investigation of the wave dynamics associated with this kind of geometry is utilized in a performance optimization study.

arbitrary prismatic channel with variable cross sectional area. The dimensionless continuity and momentum equations for the pseudo one dimensional channel flow are:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) = -\rho u \alpha_1(x)$$

$$\frac{\partial \rho u}{\partial t} + \frac{\partial}{\partial x}(\rho u u + p/\gamma) = -\rho u^2 \alpha_1(x)$$

$$\alpha_1(x) \equiv d \ln A(x)/dx$$

for $0 < x < L$ and $t > 0^+$. The variables are defined as follows

$$x \equiv x^*/L, \quad t \equiv t^* a_o/L, \quad \rho \equiv \rho^*/\rho_o, \quad p \equiv p^*/p_o,$$

$$u \equiv u^*/a_o, \text{ and } a_o^2 \equiv \gamma p_o/\rho_o .$$

The major assumption is now made, that the gas velocity is linear with the material front (blast wave front) velocity:

$$u(x,t) = xF(t) = x\dot{R}(t)/R(t)$$

where $R(t) = R^*/L$ denotes the instantaneous position of the blast front in the channel. Sedov (1959) and Zeldovich and Raizer (1966) demonstrated that the foregoing velocity distribution obtains good representation of strong explosions in air and gas expansion in vacuum. This same form is therefore assumed to be likewise applicable to the present flow field. For obvious reasons (cf the α_1 - terms in the equations of motion), similarity solutions are not sought herein.

With the linear velocity distribution, $u = x\dot{R}/R$, the continuity equation yields, after separation of variables:

$$\rho(x,t) = C_o \sum_{(n)} \frac{C_n}{R^{\lambda_n}} \left[\frac{x^{\lambda_n - 1}}{A(x)} + 1 \right], \quad 0 < x < R$$

where λ_n are discrete separation constants (or eigenvalues) and c_n are the corresponding coefficients. C_o is the integration constant. Actual determination of the allowable $\lambda_n > 0$ is based on the available boundary data. This discussion is deferred.

The momentum equation can be written as

$$\frac{\partial p}{\partial x} = -\gamma\left[\rho\frac{\partial u}{\partial t} + \rho u\frac{\partial u}{\partial x}\right] = -\gamma\rho x(\dot{F}+F^2)$$

where both the continuity equation and the velocity distribution have been implemented. Let:

$$g(t) = \dot{F} + F^2 = \ddot{R}/R$$

in general; using the available eigensolutions for the density, one obtains after integration with respect to x:

$$p(x,t) = \gamma C_1 \sum_{(n)} \frac{g_n(t)}{R^{\lambda_n}}\left[1 - I_n(x)\right]$$

$$I_n(x) \equiv \int_{(x)} \frac{x^{\lambda_n}}{A(x)}\,dx, \qquad I_n(0)=0 \quad \forall n .$$

The indefinite integrals I_n can be determined once the area variation and the eigenvalues are specified.

We turn now to derermining the blast front position in the channel, R(t) The method of Zeldovich and Raizer (1966) is followed, for the nonisentropic case. At the position x=0 we assume that the eigensolutions are related as

$$p_n^o(t) = B_n \rho_n^o(t)^\gamma$$

where B_n are the local entropy constants, which may be evaluated at t=0 from the available initial data. The eigensolutions yield

$$\rho_n^o(t) = C_o C_n / R^{\lambda_n}$$

$$p_n^o(t) = \gamma C_1 \ddot{R} / R^{\lambda_n + 1}$$

Thus, the adiabatic relationship at the x=0 point implies:

$$\gamma C_1 \ddot{R} / R^{\lambda_n + 1} = B_n (C_o C_n)^\gamma / R^{\gamma\lambda_n}$$

$$\ddot{R} - K_n R^{\beta_n} = 0, \qquad t > 0^+$$

with the initial data:

$$R(0) = R_o \text{ and } \dot{R}(0) = 0.$$

The two scalar parameters are, in general:

$$K_n \equiv \beta_n (C_o C_n)^{\gamma} / \gamma C_1$$

$$\beta_n \equiv 1 - \lambda_n (\gamma - 1)$$

Note that only two constants (e.g., C_o and C_1 or B_n and C_o) need to be specified to fully determine K_n and β_n herein; these are available from the initial data, in addition to the fluid properties γ, q^o, etc.

It should be emphasized that it is entirely possible that only a single eigensolution, corresponding to one particular value of $\lambda_n = \lambda^*$, would be physically admissible. The key to the actual choice of eigenvalues are the parameters (K_n and β_n) which would allow admissible solutions for R(t), as based on the initial data. This selection is not straightforward, as the second order (ordinary) differential equation for R(t) is nonlinear in general.

Once the eigenvalues λ_n are determined, the pressure distribution p(x,t) for $x \leq R$ can be calculated, for given A(x), gas properties, initial data, etc. This explicit expression can then be used to calculate the total impulse and the mean thrust integrals, stated earlier, for nonsteady, unisentropic nozzle expansion.

The important feature here is, that the basic fluid mechanical interactions have been incorporated, and the effect of the various physical parameters upon thrust and propulsive efficiency are clear. This is obtained (so far) without the necessity to execute 2 or 3-dimensional nonsteady numerical analysis; at most, one would have to solve numerically the nonlinear ordinary differential equation for R. The results can be readily used in an optimization study, where explicit functional relations are necessary.

6. References

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