# Incentives for accelerating the production of Covid-19 vaccines in the presence of adjustment costs<sup>\*</sup>

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#### Abstract

Delays in the availability of vaccines are costly as the pandemic continues. However, in the presence of adjustment costs firms have an incentive to increase production capacity only gradually. The existing contracts specify only a fixed quantity to be supplied over a certain period and thus provide no incentive for an accelerated buildup in capacity. A high price does not change this. The optimal contract would specify a decreasing price schedule over time which can replicate the social optimum.

# 1 Introduction

As governments grapple with the second wave in Europe, they also start mass scale vaccination campaigns, hoping to achieve herd immunity, which is the point at which a high enough percentage of the population has been vaccinated so that the virus will abate. However, vaccination takes time because increasing capacity is subject to adjustment costs. The supply of vaccines is thus limited in the short run [1].

Adjustment costs lead firms to increase capacity only gradually, which might not be optimal from a social point of view. The problem for public authorities is then to find a way to accelerate the increase in production capacity. The vaccine supply contracts were mostly concluded before the vaccines had even been fully developed, let alone approved for general use. It was thus impossible to impose a tight deadline for delivery. The Advance Purchase Agreements of the EU, two of which have been published [2, 3], thus specify only an overall

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price and tentative delivery schedules in terms of quarters, not months or weeks. When even these tentative schedules start to slip (as is the case now), the EU has little leverage to induce companies to make efforts to accelerate delivery. <sup>1</sup> We analyze the consequences of this type of contracts for the supply schedule of a vaccine and how the resulting incentives for back-loading supplies can be mitigated.

Limitations: A substantial part of the literature on vaccine policy focuses on how and whom to vaccinate, usually taking it as granted that the supply of vaccines is not a constraint [1, 4, 5, 6]. We do not consider this issue as we concentrate on the case of Covid vaccines, for which mass production had to start immediately after test trials were successful. Another, issue we do not consider is vaccine hesitancy [7], leading to doubts about whether herd immunity will be achieved if a certain proportion of the population refuses the vaccine. The immediate problem facing policy makers is the opposite, at least initially. The demand for vaccination far outstrips supply. Moreover, even if full herd immunity could not be reached, there is still a considerable benefit from every person vaccinated, which reduces potential hospitalization costs correspondingly, allowing governments in the aggregate to end lockdown measures earlier [8, 9].

Here we do not consider the issue of uncertain efficacy of vaccines and the related problem of ordering portfolios of potential candidates [8], as done by most major countries. Neither do we consider uncertainties in production costs [10].

Our analysis concentrates on the problem of ramping up production once the efficacy of a vaccine has been established [11]. The importance of this issue for the global economy has been laid out in [9].

We start by analyzing the case in which firms producing the vaccines optimize the production time path with the aim of minimizing costs under the constraint of a fixed quantity to be produced within a given period of time (and given a fixed price). We also consider the problem of building up production over time from the perspective of a social planner and show that it is equivalent to a pricing scheme that is linear in time. In this case an initially high starting price declines over time. The resulting optimal pricing scheme aligns the interests of the producer with that of the society as a whole.

# 2 Adjustment costs for ramping up vaccine production

The problem that the producers of a new product, like the Covid vaccines, face involves one key element, namely adjustment costs. It is not possible to ramp up

<sup>&</sup>lt;sup>1</sup>The Heading of the AstraZeneca contract is "ADVANCE PURCHASE AGREEMENT (APA) for the production PURCHASE AND SUPPLY OF A COVID VACCINE IN THE EUROPEAN UNION. A large part of the APA is devoted to the question that the vaccine be produced in the EU (and not elsewhere). To ensure production within the EU seems to have been the focus of the Commission at the time.

production instantaneously. Standard economic analysis takes this into account by positing that there is a cost to increasing capacity and that this cost is convex, i.e. the costs of increasing capacity are only small when the buildup is slow [12, 13]. The implication is straightforward: it will be optimal to smooth production over time.

Consider a contract in which a certain quantity  $Z_T$  is to be delivered over a period T (say one year), at a constant price  $p_0$  per unit. In this case the exact timing of the delivery, close to the start or to the end of the delivery period, does not matter and it will be optimal for the producer to minimize costs by increasing capacity only gradually over time.

Denoting the instantaneous production capacity (the number of vaccines produced per unit time, say daily) with  $z_t$ , the adjustment costs will be a function  $f(\dot{z}_t)$  of  $\dot{z}_t$ , which quantifies the speed at which production is ramped up. Overall adjustment costs are then determined by the integral of  $f(\dot{z}_t)$  over the delivery period, subject to the constraint that a total of  $Z_T = \int_0^T z_t dt$  units are produced.

In most applications [12], the convexity of the costs of adjustment is assumed to be quadratic (which would also be the result of a second order approximation). With quadratic adjustment costs, as considered here, the marginal cost of adjusting becomes linear, allowing for explicit solutions.

## 2.1 The time path for production under fixed price contracts

We start by analyzing the production path resulting from the type of contract that has been used so far, namely a fixed price against delivery of a certain quantity over a time period specified in advance. For example, the Advanced Purchase Agreement of the European Union with Curevac specifies the delivery of certain amounts of doses for the year 2021 [3], with only *tentative* delivery schedules by quarter. This implies that the firm can distribute the supply schedule over the entire year, which is nearly an eternity in terms of a pandemic costing several percentage points of GDP at each instant, threatening at the same time the lives of thousands every day. The EU contract with AstraZeneca [2] specifies only *reasonable best efforts* and the one with Sanofi [14] contains a similar formula.

We thus focus on the inter-temporal problem of increasing production capacity over time within the overall time frame given by the contract, which could be thought of representing one year. Given this time frame (and interest rates around zero), we neglect time discounting.

Formally we consider a firm which has been contracted to supply a certain amount  $Z_T$  of doses over a given period T. The marginal cost of each dose is denoted by c and is assumed to be constant once the capacity has been created.

Capacity means in this case not just the physical factory, which might have to satisfy specific requirements, but also the schooling of personnel, etc. We assume that the initial capacity is low, possibly equal to zero, but definitely not large enough to satisfy the entire order within  $t \in [0, T]$ . This implies that the firm must ramp up capacity during the contract period [0, T].

The problem for the firm is then to maximize revenues minus adjustment costs, subject to the overall production constraint:

$$p_0 \int_0^T z_t dt - a_z \int_0^T (\dot{z}_t)^2 dt - \lambda_Z \left[ \int_0^T z_t dt - Z_T \right] - c \int_0^T z_t dt \,, \qquad (1)$$

where  $p_0$  denotes the price per vaccine,  $a_z$  encodes the size of the adjustment cost, and where  $\lambda_Z$  is the Lagrange multiplier enforcing the constraint that the total production over the period [0, T] is  $Z_T = \int_0^T z_t dt$ .

We use adjustment costs in absolute, not relative terms. This means that these costs do not depend on the level of capacity already reached. Substantial effort has been devoted in the literature to the study of adjustment costs modelling them in terms of a proportional increase in capacity [12], i.e. adjustment costs that are a function of  $\dot{z}_t/z_t$ . However, this would lead to conceptual difficulties when starting the production of a new product (i.e. when  $z_0 = 0$ ), the case of Covid-19 vaccines.

This formulation (1) of transactions costs is assumed to reflect roughly real world technical constraints. That 'overnight factory constructions' are not possible is translated in the framework of equation (1) into adjustment costs that diverge to infinity. Standard variational calculus [15, 16] establishes that the stationary solution to (1) satisfies

$$2a_{z}\ddot{z}_{t} = \lambda - p_{0} - c, \qquad z_{t} = z_{0} + \gamma t + \frac{\lambda - p_{0} - c}{4a_{z}}t^{2}, \qquad (2)$$

where  $z_0$  is the initial production capacity and  $\gamma$  the speed at which production capacity increases initially.

Only the difference between price and marginal costs enters the condition 2. In the remainder we thus assume that c is equal to zero. This assumption is made only for computational convenience. Any constant marginal cost would only add a fixed amount to the overall costs of the firm because the total quantity to be produces is fixed. One could thus think of the price as representing the difference between the unit price contracted and any marginal cost of production.

Production capacity remaining at the end of the production period is worthless. This assumption can be However, our results would not change even if production capacity at the end of the contractual period were to have a value because this value would just add a constant term to the firms revenues and would thus not affect the time path for the build-up of capacity, which is our main object of analysis.

The problem that the firm faces can be reduced to minimizing the total cost of adjustment over the delivery period T, as total revenues are fixed, being equal to the price times the quantity delivered. The production schedule that minimizes the adjustment cost is to increase capacity accordingly to (2).

A constant rate of increase in production would not be optimal, on general grounds, because an increase in capacity implemented today yields higher production over the remainder of the delivery period and is thus more valuable that an increase in capacity just before the end of the delivery period. The speed at which capacity increases should thus decline over time. This intuition is born out by (2).

To be concrete, we parameterize the solution to (2) with

$$z_t = z_0 + \gamma t + \delta t^2, \qquad \delta = \frac{\lambda - p_0}{4a_z}.$$
 (3)

The production condition  $Z_T = \int_0^T z_t dt$  implies then

$$Z_T = z_0 T + \frac{\gamma}{2} T^2 + \frac{\delta}{3} T^3, \qquad \gamma = \frac{2\Delta Z}{T} - \frac{2\delta T}{3}.$$
(4)

where  $\Delta Z$  denotes the difference between the average capacity needed to fulfill the order and the initial one,  $\Delta Z = Z_T/T - z_0$ . It is assumed here that  $\Delta Z > 0$ , namely that the capacity needs to be increased. In the opposite case, when  $Z_T < Tz_0$ , the company would have to shut down part of the existing production capacity - which is not the case for Covid vaccines.

The overall production constraint (4) can be satisfied by any linear combination of  $\gamma$  and  $\delta$ . These two parameters are determined by maximizing total profit. Given that the first term in (1) is constant,  $p_0Z_T$ , one just has to minimize the adjustment costs:

$$E_{\rm adj} = a_z \int_0^T (\dot{z}_t)^2 dt = a_z \left[ \gamma^2 T + 2\gamma \delta T^2 + \frac{4\delta^2 T^3}{3} \right],$$
(5)

where  $\dot{z}_t = \gamma + 2\delta t$  has been used. The relation (4) entails that  $\partial \gamma / \partial \delta = -2T/3$ , which leads to

$$\frac{dE_{\rm adj}}{d\delta} = \frac{\partial E_{\rm adj}}{\partial\delta} + \frac{\partial E_{\rm adj}}{\partial\gamma} \frac{\partial\gamma}{\partial\delta} = \frac{\partial E_{\rm adj}}{\partial\delta} - \frac{2T}{3} \frac{\partial E_{\rm adj}}{\partial\gamma}$$
(6)

$$= a_{z} \left[ 2\gamma T^{2} + \frac{8\delta T^{3}}{3} - \frac{2T}{3} \left( 2\gamma T + 2\delta T^{2} \right) \right]$$
(7)

$$= a_z \left[ \frac{2\gamma T^2}{3} + \frac{4\delta T^3}{3} \right] = 0.$$

The first order condition for cost minimization over the choice of  $\gamma$  and  $\delta$  leads therefore to following simple relationships:

$$\gamma = -2\delta T, \qquad \gamma = 3\frac{\Delta Z}{T}, \qquad \delta = -\frac{3\Delta Z}{2T^2}.$$
 (8)

where the last two relations follow from (4). The time path  $z_t$  for the production capacity is then

$$z_t = z_0 + \frac{3\Delta Z}{T} \left[ t - \frac{t^2}{2T} \right], \qquad \dot{z}_t = \frac{3\Delta Z}{T} \left[ 1 - \frac{t}{T} \right], \qquad (9)$$

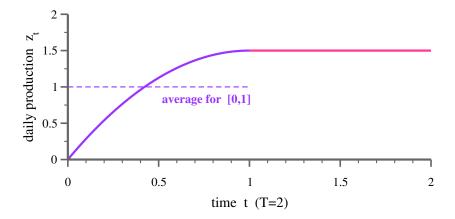


Figure 1: The time evolution of the production capacity  $z_t$ , taken here to represent the daily output. In the first period,  $t \in [0, 1]$ , an average of  $\langle z_t \rangle = 1$  is to be attained (in this example). Minimization of the adjustment costs leads to (9), which is shown. Note that  $z_{t=1}$  overshoots the average by 50%. Production per time remains constant in the second period  $t \in [1, 2]$ . During the overall delivery time T = 2 the production capacity is on the average (1+3/2)/2 = 1.25.

as illustrated in Figure 1. This implies that production capacity follows an inverted parabola, with the highest level reached just before the end of the delivery period. The *increase* in capacity starts strongly, but declines over time tending towards zero at the end of the delivery,  $\lim_{t\to T} \dot{z}_t \to 0$ . The result (9) also implies that  $\dot{z}_t$  is proportional to the missing average production capacity  $\Delta Z$ , scaling inversely with the production period T. At the end of the contract period (t=T), the production capacity will be equal to 1.5 times the one which is needed on average  $(Z_T/T \text{ when } z_0 = 0)$ .

Note that the cost minimizing production path  $z_t$  does not depend on the overall value of the order since the price  $p_0$  does not influence the parameters of the differential equation (9). The reason is that  $p_0$  enters the Lagrange multiplier of the equation of motion (4) only through the difference  $\lambda - p_0$ .

The key corollary from the above considerations, regarding the effects of adjustment costs, is then:

The level of the price does not influence the speed at which production is increased – when the price is constant.

Higher prices allow the producer to obtain larger profits, however without providing incentives to accelerate the buildup of the capacity. We have not considered explicitly the cost of developing the vaccine, which would add a constant term to the costs for the firm. But this constant term would also not have any influence on the speed at which production is increased since it represents just a sunk cost when the firm starts to ramp up production.

#### 2.2 Adjustment costs scaling

We have established so far that a constant price does not affect the time path of production, but the firm will accept the contract only if total revenues,  $Z_T p_0$  compensate all costs costs,  $E_{adj}$ , which can be obtained by substituting (8) into (5).

$$E_{\rm adj} = 3a_z \frac{\Delta Z^2}{T} = 3a_z \frac{(Z_T - z_0 T)^2}{T^3}, \qquad (10)$$

Total adjustment costs are linear in the adjustment cost parameter,  $a_z$ . For  $z_0 = 0$ , ceteris paribus, they fall with the cube of the time the firm has to fulfill the entire order. A positive level of initial production  $z_0 > 0$  helps to reduce adjustment costs and this effect increases with the length of the time period given for delivery, T. These scaling relations hold for fixed overall production  $Z_T$ .

The equation 10 represents also the minimum total expenditure public authorities would have to sustain in a market in which firms compete for vaccine orders. (Development costs were separately financed.) The EU has concluded contacts with 6 suppliers of vaccines which appeared to have a realistic chance of success in 2020. There was thus some, but also certainly not perfect, competition across different vaccine producers.

We do not take a stance on how competitive the vaccine market is and whether the prices actually paid reflected mainly costs. However, one can still recover from the expression for the total adjustment costs a lower bound; i.e. the unit price needed to induce a firm to accept the contract.<sup>2</sup> This is given for  $z_0 = 0$  by:

$$\frac{E_{\rm adj}}{Z_T} = 3a_z \frac{Z_T}{T^3} \tag{11}$$

A first corollary of (11) is that, for a given amount ordered,  $Z_T$ , the unit cost, and thus the shadow reservation price, increases with the cube Halving T requires an 8-fold higher price to compensate for rapidly increasing adjustment costs. if a higher number of doses has to be delivered by the same time. The price would still have to increase four times price when halving both the delivery time T and the total amount requested  $Z_T$ , thus keeping average delivery intensity constant. Moreover, given T, a larger order requires a higher unit price. The underlying reason is that capacity has to be increased faster when starting from zero.

We note that the overall delivery time T is not the same as the average delivery delay  $t_{\text{deliver}}$ , over the life-time of the product, which is given by

$$t_{\text{deliver}} = \frac{1}{Z_T} \int_0^T z_t \, t \, dt = \frac{1}{Z_T/T} \left[ \frac{z_0}{2} + \frac{5\Delta Z}{8} \right] T \,. \tag{12}$$

 $<sup>^{2}</sup>$ We recall that the price was defined as the difference to the marginal costs of production. The shadow prices discussed here should thus be increased by marginal costs. The same argument can also be applied if the capacity build up to satisfy this one-time order has a scrap value. This value will not affect the time path of production, but it will affect the price at which the firm can break even.

With  $\Delta Z = Z_T/T - z_0$  this result implies that for  $z_0 = 0$ , i.e. when the initial capacity is zero and  $\Delta Z = Z_T/T$ , the mean delivery delay will be 5T/8. Instead, when  $\Delta Z = 0$ , viz when production is constant, the average delay would be T/2. The later case holds when the initial production capacity is sufficient to fulfill the entire order over time, which is however not the case for Covid-19 vaccines.

#### 2.3 Social costs of delay vs. adjustment costs

For society the value of a dose depends importantly on the time it is delivered. Early delivery helps to avoid infections and allows for an earlier lifting of lockdowns. This implies that any delivery which does not occur today imposes an opportunity cost for society which is due to the economic loss of prolonged lockdowns and more infections. Each early dose thus delivers a flow of benefits in terms of avoided costs which is proportional to the time it arrives. This benefit does not materialise only when herd immunity has been reached. Every person vaccinated will reduce the potential medical costs from an infection. We thus assume that the opportunity cost to society of the 'no vaccination' status quo is k per unit of time, where k parametrizes the per unit costs of lost output, due to a continuing lockdown. Assuming that the total amount ordered,  $Z_T$ is sufficient to stop the pandemic, the cost is reduced pro rata the part of the population which has been vaccinated,  $\frac{z_t}{Z_T}$ 

The aim for society should be to minimize the sum of social opportunity costs of delay and the adjustment costs in ramping up vaccine production. For the case of a contract with a fixed price, the total adjustment cost were calculated in (10). This can be combined with the time path of capacity (which is proportional to the number of people vaccinated each period) to yield:

The economic costs are proportional to the relative number of people yet not vaccinated,

$$k \int_{0}^{T} \left( 1 - \int_{0}^{t} \frac{z_{t'}}{Z_{T}} dt' \right) dt = k \int_{0}^{T} \left( 1 - \frac{3}{2} \frac{t^{2}}{T^{2}} + \frac{1}{2} \frac{t^{3}}{T^{3}} \right) dt$$
$$= \frac{5}{8} kT, \qquad (13)$$

where (9) has been used, together with  $z_0 = 0$ . Over the same period the costs would be kT if nobody would be ever vacinated, which implies that ordering vaccines at a fixed pice leads to a reduction of opportunity costs of 3/8 already within the delivery period, together with 100% reduction afterwards.

Overall social costs  $E_{\text{social}}$  are the economic costs (13) together with the adjustment costs (10),

$$E_{\text{social}} = \frac{5}{8}kT + 3a_z \frac{Z_T^2}{T^3},$$
 (14)

where (10) and (12) have been used for the case  $z_0 = 0$ . The delivery period T minimizing the sum of social and adjustment costs is hence given by

$$T_{\rm opt} = \left[\frac{72Z_T^2}{5}\frac{a_z}{k}\right]^{1/4}.$$
 (15)

Using the solution for the optimal time (15) in the expression for the unit adjustment costs (11) yields a solution for the price at which the firm would be willing to supply:

$$\frac{E_{\text{adj}}}{Z_T} = 3a_z \frac{Z_T}{T^3} = 3a_z Z_T^2 \left[\frac{5k}{72a_z Z_T^2}\right]^{3/4} = 3a_z^{\frac{1}{4}} \left[Z_T\right]^{-1/2} \left[\frac{5k}{72}\right]^{3/4}.$$
 (16)

This implies that the unit cost for the firms increases steeply with the total amount ordered (exponent 1/4), and almost linearly (exponent 3/4) with the social cost factor k. The unit cost at which firms would supply (in the context of a fixed price contract) thus does not depend only on the *ratio* of these two parameters. We will return to the relative size of adjustment and social costs in Sect. 3.1.

## 2.4 Societal gains from vaccination and asymmetric information

The unit costs in (16) reflect only the minimum needed for the firm to break even. It does not reflect the amount the authorities should be willing to pay, which should be function of the total societal costs avoided. Without a vaccine the pandemic would continue until most people have been affected and natural herd immunity has been reached. With continuing NPIs, this would not happen in an uncontrolled exponential growth, but gradually over time. For example, in the US, at least 8 percent of the population has been infected (in the sense that 8 percent had a positive test result) over one year. This implies that it would take several years under similar conditions for natural herd immunity to be reached. The gain to society from the availability of a vaccine should thus be measured against a baseline of the costs continuing not until T, but until until natural herd immunity is reached,  $T_{nhi}$ . The total gain to society would thus be equal to k times the period of zero cost after full vaccination (which is equal to  $T_{nhi} - T$ ) plus the gains reaped during the vaccination period.

The latter can be calculated assuming that the delivery time, T, is set by the authorities according to equation (15) the relationship. Substituting this back into the expression for the overall social cost (14) yields an expression for the 'minimum social cost', i.e., the social costs that remains even if the time has been set so as to minimize social costs:

$$E_{\min \text{ social costs}} = T\left[\frac{5}{8}k + 3a_z \frac{Z_T^2}{T^4}\right] = Tk\left[\frac{5}{8} + \frac{15}{72}\right] = \frac{5Tk}{6}.$$
 (17)

This implies that (with a fixed price contract), only one sixth of the cost of the pandemic that arises during the period of increasing production to vaccinate the population can be avoided.

SocialBenefit = 
$$k \left[T_{nhi} - T\right] + \frac{Tk}{6} = Tk \left[\frac{T_{nhi}}{T} - \frac{5}{6}\right],$$
 (18)

The ratio of social benefits to overall expenditure needed to obtain the necessary amount of vaccines in the optimal period can now be calculated using (9) together with (15) as:

$$\frac{\text{SocialBenefit}}{\text{Cost of order}} = kT \frac{\frac{T_{nhi}}{T} - \frac{5}{6}}{3a_z \frac{Z_T^2}{T^3}} = \frac{24}{5} \left[ \frac{T_{nhi}}{T} - \frac{5}{6} \right].$$
(19)

This implies that the social benefits should be a multiple of the cost of placing the order (provided, of course, that the time needed to reach natural herd immunity would be longer than the time needed to reach this goal through vaccination). For example, social benefits would be twenty times larger than the cost if vaccination would reduce the time of the pandemic by one fifth.

This potentially very large relative difference between social benefits and the private shadow price would become important if there is asymmetric information. The authorities would be willing to pay a much higher price than would be necessary to induce companies to supply the vaccine.

We have so far considered only contracts which specify a fixed price. The optimal contract time calculated in (15) above constitutes a second best, because it is subject to this constraint. We now turn to the optimal contract design when this constraint is lifted.

# 3 Optimal time-varying pricing

Using the expression for the opportunity costs of delay introduced above in (13), the general social planner problem, which is not constrained by a fix price contract, is to minimize the sum of the costs of an ongoing lockdown and the adjustment costs that are necessary to accelerate production. The end point, T, represents the point in time when herd immunity has been reached, i.e. when a high per percentage of the entire population has been vaccinated. At this point economy would be fully back to normal and the costs parametrized by k no longer arise. It is usually assumed that for Covid-19 herd immunity requires that about 70 percent are vaccinated. We approximate this by normalising  $Z_T = 1$ .

Denoting total social costs by  $W_{\text{social}}$ , the social planner takes in account the opportunity costs of gradual delivery, which are proportional to the time one waits for the delivery of the vaccine:

$$W_{\text{social}} = k \int_0^T t z_t dt + a_z \int_0^T (\dot{z}_t)^2 dt - \lambda \left[ \int_0^T z_t dt - 1 \right] \,. \tag{20}$$

It can now been shown that the problem for the social planner can be made isomorphic to that of the firm. The key variable for the firm is the price, or revenue per unit produced. In a fixed price contract this price does not vary with the time the vaccine is delivered. This can be changed if the authorities offer a time varying price for example one which declines from a certain initial level. With a price  $p_t$  that is variable over time, the total revenues of the firm are given by:

$$Revenue = \int_0^T p_t z_t dt \,. \tag{21}$$

For both, the social planer and the firm, the problem has to be solved taking into account adjustment costs. The social optimum of (20) can be reached if the price path facing the firm coincides with the minimization of the pandemic costs, i.e. if

$$p_t = p_0 - kt \,, \tag{22}$$

where  $p_0$  now denotes the 'base price', which diminishes linearly over time. The problem facing the firm then becomes to maximize total revenues minus adjustment costs:

$$\int_{0}^{T} (p_0 - kt) z_t dt - a_z \int_{0}^{T} (\dot{z}_t)^2 dt - \lambda \left[ \int_{0}^{T} z_t dt - 1 \right], \qquad (23)$$

which can be rewritten as:

$$-k\int_{0}^{T} tz_{t}dt - a_{z}\int_{0}^{T} (\dot{z}_{t})^{2}dt - (\lambda - p_{0})\int_{0}^{T} z_{t}dt + \lambda.$$
(24)

Comparing equations (20) and (24) shows that they lead to the same solution, vzi to the time path for  $z_t$ . Note that the sign of the Lagrange parameter  $\lambda$  changes, but this is irrelevant. The firm maximizes the difference between revenues and adjustment cost, with unit revenues declining linearly over time. Society minimizes total costs, which comprise the same adjustment costs, but taking also into account that the costs of delayed delivery are linear in time. With the pricing schedule (22), equations (20) and (23) represent hence the same problem, except for the constant term  $p_0$ , which implies that they have the same solution  $z_t$ . The size of the initial price,  $p_0$ , has no consequences for the decision of the firm regarding how quickly to increased capacity.

The implication is that the pricing schedule (22) can induce firms to adopt the speed of increase in production capacity which is also optimal from a social point of view. There is thus a way to align private and public interests by specifying a pricing schedule which mimics the social cost of a continuing pandemic. The base price  $p_0$  determines, as before, whether the firm makes a profit or a loss, taking into account adjustment costs. The optimal contract thus involves a base price which allows the firm to break even and a premium for early delivery, which declines over time.

We also note that the pricing schedule (22) remains optimal from a social welfare point of view even if there is uncertainty about adjustment costs, which would affect the optimal schedule in exactly the same way for a cost minimizing firm as for a social planner. Given that k can be assumed to be large, because social costs affect the entire economy, this strategy may lead to a high, but also quickly declining premium. With such a pricing schedule there would be

no need to specify intermediate delivery dates (as done in existing contracts). Firms would have the incentive to ramp up production as quickly as required by society.

### 3.1 Orders of magnitude for social costs

The orders of magnitude of the social cost of a continuing pandemic can be estimated using the available data on the economic cost of the pandemic so far, which have been around 4-5 percent of GDP. Reaching herd immunity thus allows society to avoid costs equivalent to 4-5 percent of GDP, and even when including value of life costs [17]. This would mean that the avoided economic costs per vaccinated person would be equal to 4-5 percent of GDP per capita. Each dose would then be worth 2-2.5 percent of GDP per capita if two doses are needed, which would amount to 1200-1500 USD for the US and 1000-1250 for Germany. This would be between 66 and 80 times the price of 15 (euro) which has been reported for a single dose of the Pfizer/Biontech vaccine [18].

Another approach to determine the value of a vaccination relies on surveys of the willingness to pay (WTP) expressed in standard surveys used to estimate the value of other vaccines. One study [19] concludes that the social valuation of vaccination is about 1.1 percent of the per capita gross domestic product (GDP). This would be equivalent to about 600 USD per dose for the US or 500 USD for Germany. These values constitute a lower threshold as the social value of a vaccination is likely to be substantially larger than the private value, because vaccinated individuals no longer transmit the disease to others.

The estimate of the overall cost of the Covid pandemic presented in [9] suggests a similar order of magnitude, but expressed in total amounts. It is estimated that the global total cost of the Covid pandemic is about 16 trillion USD (of which about one half is due to medical cost and the value of lives lost), which could be avoided through 6 billion vaccinations resulting in a social value of about 2600 USD per vaccination (1300 per dose if two are needed for immunity).

These estimates indicate that the social value for the delivery of a vaccine today should be very high, with 1500 USD as an upper bound, and 500 USD as a lower bound. The price a society should be willing to pay for a dose available immediately should be consequently between these two values. However, the price would decline rapidly over time, tending to zero towards the end of the delivery period (always relative to marginal costs).

The hitherto fixed prices paid by governments for Covid-19 vaccines have been made public only partially [18, 20], but they are generally in the region of 15-30 USD per dose. This is more than one order of magnitude below the social value for an intermediate delivery (both for the lower and the upper bound).

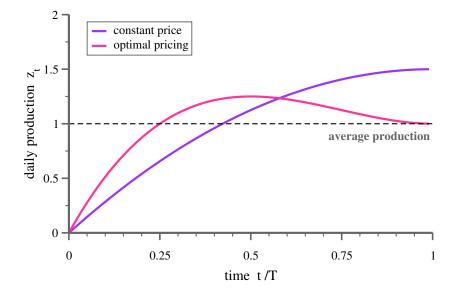


Figure 2: The time evolution of the production capacity  $z_t$ , as for Fig. 2, but with optimal pricing (22), which leads to the production timeline  $z_t$  as determined by (28) and (31). Shown is the case  $\kappa = 4\Delta Z/T^3$ , see (32), for which the final production level equals the average production  $\Delta Z$ . Note the temporary overshooting. As a comparison, the result for constant pricing,  $\mu = 0$  in (22), is also given.

### 3.2 Optimal time path for capacity

The evolution of the production capacity  $z_t$  optimizing (23) is determined by

$$2a_{z}\ddot{z}_{t} = \lambda - p_{0} + kt, \qquad z_{t} = z_{0} + \gamma t + \frac{\lambda - p_{0}}{4a_{z}}t^{2} + \frac{k}{12a_{z}}t^{3}, \qquad (25)$$

where  $\lambda_Z$  is again the Lagrange multiplier enforcing the constraint that the total production over the period [0, T] is:

$$Z_T = 1 = \int_0^T z_t dt = z_0 T + \frac{\gamma T^2}{2} + \frac{(\lambda - p_0)T^3}{12a_z} + \frac{kT^4}{48a_z}.$$
 (26)

Minimizing adjustment costs implies that at the end of the production period it does not pay to build up further capacity. This implies from (25) that:

$$0 = \dot{z}_t \Big|_{t \to T} = \gamma + \frac{\lambda - p_0}{2a_z} T + \frac{k}{4a_z} T^2 .$$
 (27)

The quantitative importance of a declining price schedule can be illustrated by parameterizing  $z_t$  as

$$z_t = z_0 + \gamma t + \delta t^2 + \kappa t^3, \qquad \delta = \frac{\lambda - p_0}{4a_z}, \qquad \kappa = \frac{k}{12a_z}.$$
 (28)

One obtains with delivery condition (26) that

$$\gamma = \frac{2}{T^2} - \frac{2\delta T}{3} - \frac{\kappa T^2}{2}, \qquad (29)$$

where the initial production capacity,  $z_0 = 0$ , has been set to zero. We can now use the condition that at the end of the period it does not pay to build up further capacity:

$$0 = \dot{z}_t \Big|_{t \to T} = \gamma + 2\delta T + 3\kappa T^2 \,, \tag{30}$$

which reduces to (8) for  $\kappa = 0$ . From (29) and (30) one finds

$$\gamma = \frac{3}{T^2} + \frac{3\kappa T^2}{4}, \qquad \delta = -\frac{3}{2T^3} - \frac{15\kappa T}{8}.$$
(31)

Of interest is in particular the choice  $\kappa = 4/T^4$ , which leads to

$$\gamma = \frac{6}{T^2}, \qquad \delta = -\frac{9}{T^3}, \qquad z_t|_{t=T} = \frac{1}{T}.$$
 (32)

This result implies that the company falls back to the average production at the end of the delivery period when  $\kappa = 4/T^3$ , as illustrated in Fig. 2. At this point production is built up at twice the speed resulting from constant pricing, viz when  $\kappa = 0$ .

### 3.3 Pandemic costs

The general public is directly affected in particular by the first term of the social costs (20), which determines effectively when and how intensive the vaccination campaign is. This term, the 'pandemic cost', is given by

$$W_{\text{pandemic}} = \int_{0}^{T} z_{t} t dt = \int_{0}^{T} \left( z_{0} t + \gamma t^{2} + \gamma t^{3} + \kappa t^{4} \right) dt$$
$$= \frac{z_{0}}{2} T^{2} + \frac{\gamma}{3} T^{3} + \frac{\delta}{4} T^{4} + \frac{\kappa}{5} T^{5} .$$
(33)

Using the explicitly expressions (31), and  $z_0 = 0$ , the pandemic costs take the form

$$W_{\text{pandemic}} = \left(\frac{3}{T^2} + \frac{3\kappa T^2}{4}\right) \frac{T^3}{3} - \left(\frac{3}{2T^3} + \frac{15\kappa T}{8}\right) \frac{T^4}{4} + \frac{\kappa}{5}T^5$$
$$= \frac{5}{8}T - \frac{3}{160}\kappa T^5.$$
(34)

For the choice  $\kappa = 4/T^4$  discussed above, see (32), for which the production capacity  $z_t$  returns to the average value at the end of delivery period  $t \to T$ , the pandemic costs per time,  $W_{\text{pandemic}}/T$ , become 97/160.

# 4 Conclusions

Our analysis starts from the observation that delays in the availability of vaccines are very costly for society. A dose delivered one quarter later is substantially less valuable than a dose delivered today. The costs per unit of time remain high as the pandemic continues and governments are forced to implement lockdowns that depress the economy. However, the resulting urgency to speed up delivery is not recognized in the existing contracts, which specify mostly only a fixed quantity and an overall time frame, typically the entire year of 2021. In the absence of incentives to produce early, firms will tend to minimize adjustment costs, viz the costs resulting from ramping up production fast. It will then be preferable for firms to increase production capacity only gradually.

Our analysis shows that the lack of incentives to produce early does not derive from a potentially low *level* of the price offered to companies but on its *time path*. With the existing, fixed price contracts, a dose delivered the subsequent quarter yields the same revenue for the producer as a dose delivered today, but for society there is a huge difference. The practical problem is then how to provide incentives for early delivery.

A better contract would have the price fully variable over time. We show that it is straightforward to design an optimal contract, which aligns the time paths of the price with that of the social value of a vaccination. In this case linearly decreasing price schedules replicate the social optimum.

From our perspective there is a clear policy conclusion: Supply contracts for vaccines should contain incentives for accelerated production. Vaccines delivered early should command a higher price.

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