How fast must vaccination campaigns proceed in order to beat rising Covid-19 infection numbers?[*]

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Abstract

We derive an analytic expression describing how health costs and death counts of the Covid-19 pandemic change over time as vaccination proceeds. Meanwhile, the disease may continue to spread exponentially unless checked by Non Pharmacological Interventions (NPI). The key factors are that the mortality risk from a Covid-19 infection increases exponentially with age and that the sizes of age cohorts decrease linearly at the top of the population pyramid. Taking these factors into account, we derive an expression for a critical threshold, which determines the minimal speed a vaccination campaign needs to have in order to be able to keep fatalities from rising. Younger countries with fast vaccination campaigns find it substantially easier to reach this threshold than countries with aged population and slower vaccination. We find that for EU countries it will take some time to reach this threshold, given that the new, now dominant, mutations, have a significantly higher infection rate. The urgency of accelerating vaccination is increased by early evidence that the new strains also have a higher mortality risk [1]. We also find that protecting the over 60 years old, which constitute one quarter of the EU population, would reduce the loss of live by 95 percent.

1 Introduction

After a first and second wave, Europe faces in the spring of 2021 a rapidly spreading third Covid-19 outbreak. There is an ongoing race between mass scale vaccination campaigns and the disease, which continues to evolve and to

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spread [2]. The emergence of a new and more infectious strand [3] means that strict containment measures need to be put in place to prevent exponential growth of cases, hospitalizations and fatalities. At the same time, vaccines have become available, with vaccination campaigns protecting a growing proportion of the population, slowing and eventually also stopping the spread of the SARS-CoV-2 virus [4]. The ultimate aim is 'herd' or population immunity, when a high enough percentage of the population has developed immunity through infection or vaccination.

However, vaccination takes time, mostly due to the limited availability of the vaccine, in part also due to bottlenecks in distribution and implementation. In this respect we point out that the production of SARS-CoV-2 vaccines is observed to increase approximately linearly with time and that this observation is in line with the prediction that firms processing batch orders minimize their adjustment costs when ramping-up production linearly [5].

For policy makers, a core question is whether it may be possible to lift NPIs at least partially already comparatively early in the vaccination campaign. To this respect we concentrate here not on the number of infections, but on the medical costs, which we take to be proportional to the number of lives at risk. The number of fatalities and hospitalizations for Covid-19 tend to be highly correlated. Moreover, the number of Covid-19 related fatalities constitute a key determinant for the imposition of lock-downs and other NPIs which come with severe economic and social cost in terms of lost output and employment.

The central issue for policy makers today, especially in Europe, is at what point the vaccination campaign has acquired sufficient speed to overcome the increase in infections, which occur in many countries despite partial lock-downs.

To determine this, three factors need to be taken into account:

- The age-dependency of the case fatality rate, which has been established to increases approximately exponentially with age [6].

- The population structure for the elderly, which is to first order linear at the top, which means that the size of age cohorts increases gradually top-down from the maximum age (about 100 years).

- The functional dependency of daily vaccinations rates, which are observed to increase in most countries approximately linearly over time. Modulo organizational problems and reserves for the second jab, vaccination rates are determined in the end by the availability of vaccines.

These three building blocks, which constitute the foundations of our analysis, are laid out separately below.

2   Modeling framework

2.1   Age-dependency of the mortality risk

It has been widely documented that the risk to die from a Covid-19 infection raises strongly with age. A meta-study suggests an exponential relationship [6],
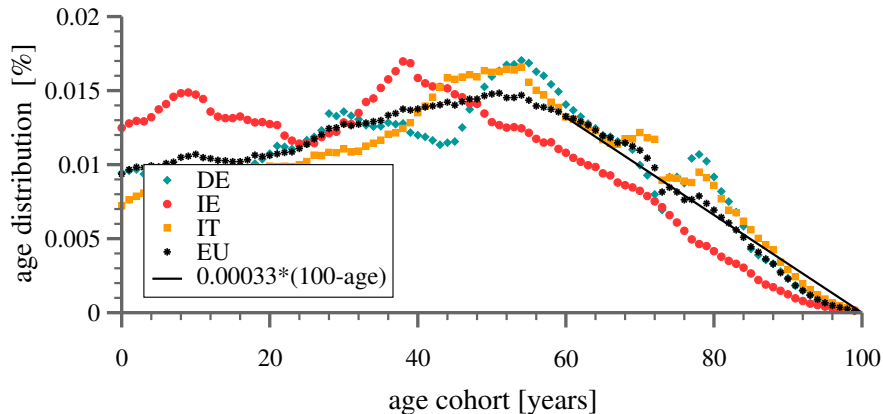
Figure 1: **Age distribution of selected EU countries.** Fractional age cohorts for Germany (DE), Ireland (IE), Italy (IT) and EU27 (EU). Also shown is a linear interpolation for the EU data (black line), for ages 60 and above. Data from [7].

which can be parameterized as

$$\text{IFR} \approx 0.01 \mathrm{e}^{-7.529 + 0.121*a} \sim \mathrm{e}^{a/a_0} \qquad a_0 = 8.26 \tag{1}$$

where $a \in [0, 100]$ is the age cohort. The infection fatality rate $\text{IFR} \in [0, 1]$ is very high for $a = 100$, namely $\text{IFR}(100) = 0.93$.

The constant a_0 denotes the half live age difference in terms of mortality. To be more precise, for an age difference of 8.26 years the risk increases by a factor of 2.78 ($= \ln(1)$). The risk doubles for an age difference of 5.7 years.[1]

2.2 Age pyramid

The age pyramid for a range of selected countries is presented in Fig. 1, where the age cohorts are given in percentage, viz relative to the entire population. One notices that the age pyramid closes quadratically at the top. The relevant range for the quadratic dependency is however somewhat restricted, applying only for ages about 85 and above.

Most Covid-19 vaccination campaigns currently in place follow, to a varying extend, top-down strategies [9]. In order to quantify the impact on expected fatality counts we use a simple linear approximation for age pyramids, as included in Fig. 1. This approximation is intended as an overall fit to ages 60 and above.

In the following we set the maximum age to zero, counting down from 100. The actual age is then $100 - x$. The age density, denoted by $\rho(x)$, varies by

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[1]Statistics from the CDC of the US show that the over 85 years old have a Covid mortality rate 7900 times higher than that of the 5-17 years old. This translates also into a doubling age difference of about 8 years ($= (85-11)/\ln(7900)$ [8].

cohorts. Countries with population pyramids closing linearly at the top are described by

$$\rho(x) = px, \qquad \int_0^{\delta A} \rho(x)\,dx = v, \qquad v = \frac{p(\delta A)^2}{2}, \qquad (2)$$

where v is the number of people (relative to the total population) vaccinated top-down to an age difference δA.

Fitting to aggregate European demographic data yields $p \approx 0.00033$, as shown in Fig. 1, with little difference across the major EU member countries. This value for p refers to the case that $\rho(x)$ is measured relative to the total population, in terms of percentiles of cohorts by year. The linear approximation holds down well to 60 years. Below this age the size of the cohort no longer increases (and even falls in some countries, like Italy). Here we concentrate on the age cohorts from 60 years up, which are the ones subject to the highest mortality risk, constituting the largest proportion of the overall loss of life. The case of Germany illustrates this proposition. Taking into account the combined effect of (1) and the age distribution, as presented in Fig. 1, one finds that about 1.5 million above an age of sixty die in the hypothetical scenario that the entire population would be eventually infected with the SARS-CoV-2 virus. In contrast, the fatality count would include only 75 thousand below sixty, a factor twenty less. We thus feel justified concentrating our analysis on the age cohorts above 60, for which the population pyramid is approximately linear.

The 'over 60' account for about 26 percent of the total population of the EU, with their shares ranging from 20 percent in the case of Ireland and 29 percent in the case of Italy. This implies that vaccinating about one fourth of the population will avoid 95 percent of the fatalities (19/20). This calculation based solely on age represents of course an approximation.

Due to vaccine hesitancy the uptake among the elderly could be less than 100 percent. But these factors are also present among all age groups, reducing thus the overall benefit from a vaccination campaign, but not necessarily the advantage of age-sensitive vaccination. Vaccine hesitancy is in particular likely to be lower among the elderly, implying that the share of the benefits from offering vaccination to the elderly first might be even higher than the 95% suggested on demographic considerations alone. A factor suggesting otherwise may however be 'long Covid' [10].

There is also evidence that immunity wanes more quickly at higher ages [11], implying that the re-infection risk is higher for the elderly. This effect plays out however on a time scale beyond that of most vaccination campaigns.

2.3 Vaccination campaigns

It is not possible to vaccinate the entire population instantly, because vaccines have to be first mass-produced and then distributed. This is illustrated in Fig. 2, where the daily vaccination rates are shown for a range of selected countries. Daily rates may vary, in particular for smaller countries, when larger batches
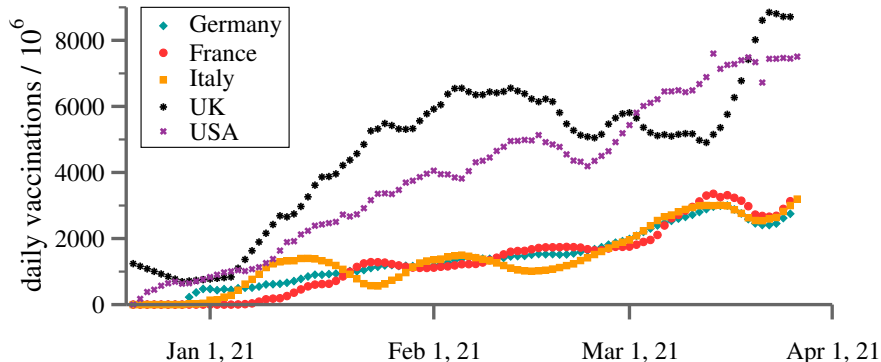
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Figure 2: **Growth of daily vaccination rates.** Per million daily vaccinations in Italy, France, Germany, the United Kingdom (UK) and the United States (USA). The growth is roughly linear, modulo in part substantial fluctuations. Data smoothed over seven days, from [12].

are delivered from abroad. The availability of imports explains also the faster vaccination campaign of the UK. But for larger units, like the EU or the US, the trend is linear. Overall, vaccination rates can be expected to track deliveries, with eventual organizational problems leading only to temporary delays. EU member countries agreed to place joint Advance Purchase Agreements with the deliveries distributed on a per capita basis [5]. As a consequence, the vaccination curves for EU member countries all follow the same trend.

Over the course of several months, the daily vaccination rates shown in Fig. 2 raise roughly linearly, in line with production capacities. This linear ramp-up has been predicted [5]. The reason is that Covid-19 vaccine were ordered ahead of their approval in large batches. But ramping up production capacities implies large adjustment costs. Minimizing these adjustment costs, subject to fulfill the order within a certain time period leads then to a linear ramping-up of production capacities [5].

Given these considerations, and the data presented in Fig. 2, we assume that the number of daily jabs, viz the vaccination rate, increases linearly. The fraction of the population v vaccinated top down increases then with the square of time t,

$$v = v(t) = \frac{1}{2}\left(\frac{t}{t_0}\right)^2 = \frac{p}{2}(\delta A)^2, \qquad \delta A = \frac{t}{t_0\sqrt{p}} \equiv a_0 g_v t. \qquad (3)$$

The parameter t_0 denotes the time needed to vaccinate one half of the population. Given the increase of cohort sizes with age, one finds that the age of the youngest cohort that can be fully vaccinated, denoted δA, falls linearly over time, see (2). The factor a_0 in the last definition is the characteristic age determining the exponential functionality of the IFR, as defined by (1).

An order of magnitude estimate for the length of vaccination campaigns, t_0, can be evaluated from available data. For example, in Israel it took about

10 weeks (from the beginning of January to mid-March) to fully vaccinate one half of the population, resulting in an estimate of $t_0 = 10$ (weeks). In the EU, only about 5 percent of the population has been fully vaccinated over the same period corresponding to an estimate of $t_0 = 10\sqrt{10} \approx 32$ (weeks).

In our framework 'vaccinated' implies full immunity, which is attained for most Covid-19 vaccines only after the second jab. For simplicity we abstained here to model reduced levels of immunity, like 95%, which would lead only to higher order corrections. Note that only a certain fraction of all jabs, typically of the order of 50%, are administered following a strict age criterion.

The time needed to vaccinate the entire population, i.e. to the point $v = 1$, is equal $\sqrt{2}t_0$. The parameter t_0 thus does not denote the full length of the vaccination campaign, but the time needed to vaccinate 50% of the population. At that point more than 99% of the fatalities can be avoided and NPIs can be lifted. t_0 provides thus a good parametrization of the effective length of the vaccination campaign.

3 Flattening the health cost curve

Putting the three basic elements – the exponential age-dependency of the case fatality rate, the linear functionality of population pyramid, and the linear in increase of daily vaccinations rates – together, we proceed to calculate the impact of a vaccination campaign on mortality rates.

We concentrate on the *growth* of health costs (here proxied by fatalities) because the key concern for policy makers remains to 'flatten the curve', i.e. to prevent an explosive increase in hospitalizations which could overwhelm health systems.

3.1 Putting the building blocks together

We assume that full vaccination provides a high level of protection against severe illness and death, as confirmed not only by trial data [13], but also by real world application [14] [15]. For our model we assume furthermore that vaccination is allocated strictly by age, starting with the oldest. In practice the situation is more complicated. Firstly, because a substantial fraction of the available vaccine is reserved in most countries for potential spreaders [9], independent of their age. Secondly, one needs to distinguish between people having received one or two shots. Both effects could be incorporated into the framework developed here. In order to clarify the mechanisms at work, we study in the following the idealized situation that 'vaccinated' implies full protection.

People belonging to the elderly group, as specified above, have the risk $R_v \in [0, R_{\max}]$ to die from a Covid-19 infection, with R_v being determined by

$$\frac{R_v}{R_{\max}} = \frac{p}{v} \int_0^{\delta A} x \, e^{-x/a_0} dx = \frac{pa_0^2}{v} \left[1 - \left(1 + \frac{\delta A}{a_0} \right) e^{-\delta A/a_0} \right], \qquad (4)$$

where the exponential $\exp(-x/a_0)$ describes the age dependency of the IFR. For (4) we used $a_0^2 d[(x/a_0 + 1)\exp(-x/a_0)]/dx = -x\exp(-x/a_0)$. As a function of the population density parameter p we then have

$$\frac{R_v(p)}{R_{\max}} = \frac{pa_0^2}{v}\left[1 - \left(1 + \frac{\delta A}{a_0}\right)\mathrm{e}^{-\delta A/a_0}\right], \qquad \delta A = \sqrt{2v/p}, \qquad (5)$$

where v is fixed when comparing two countries with different p. Note that $R_v(p)$ is in general a non-linear function of p, with the dominant term being however a linear contribution.

3.2 Small vaccination limit

For the limit $\delta A \to 0$ one finds

$$
\begin{aligned}
(1 + \tilde{A})\,\mathrm{e}^{-\tilde{A}} &\approx (1 + \tilde{A})\left(1 - \tilde{A} + \frac{1}{2}(\tilde{A})^2 - \frac{1}{6}(\tilde{A})^3\right) \\
&= 1 - \frac{1}{2}(\tilde{A})^2 + \frac{1}{3}(\tilde{A})^3,
\end{aligned}
\qquad (6)
$$

where we introduced the abbreviation $\tilde{A} = \delta A/a_0$. Together with the prefactor (5), we then have

$$\frac{R_v}{R_{\max}} \quad\rightarrow\quad \frac{pa_0^2}{v}\left(\frac{1}{2}\left(\frac{\delta A}{a_0}\right)^2 - \frac{1}{3}\left(\frac{\delta A}{a_0}\right)^3\right) = 1 - \frac{2}{3}\left(\frac{\delta A}{a_0}\right), \qquad (7)$$

when making use of $\delta A = \sqrt{2v/p}$. The oldest have in our framework the risk $R_{\max}$ to die when infected, which is consistent with the infection fatality rate being $R_{\max}\exp(-x/a_0)$.

3.3 Balancing condition

Health costs are determined by the per-person risk to die from an infection, times the probability to catch the virus. We multiply (5) with the fraction of vaccinated, v, in order to obtain an expression for total, and not for relative numbers. The medical costs C^{med} per time unit can then be written as

$$C^{\mathrm{med}} = c_0^M\left(1 + \frac{\delta A}{a_0}\right)\mathrm{e}^{-\delta A/a_0}\,I(t), \qquad (8)$$

where the number of infected, $I(t) \in [0,1]$, is defined relative to the total population. The expected costs of intensive care is c_0^M for the case that nobody has yet been vaccinated. With $\delta A = \sqrt{2v/p}$ the representation

$$C^{\mathrm{med}} = c_0^M\left(1 + \sqrt{\frac{v}{v_0}}\right)\mathrm{e}^{-\sqrt{v/v_0}}\,I(t), \qquad v_0 = \frac{a_0^2 p}{2} \qquad (9)$$

is obtained. The reference vaccination fraction v_0, as defined above, can be estimated to be $8.26^2 \cdot 0.00033/2 \approx 0.011$ for a typical EU country, viz roughly one percent.

As argued above, a key concern for policy makers remains 'to flatten the curve'. This means that the key aspect is evolution of medical costs over time, as expressed by the time dependence $\delta C = (dC^{\mathrm{med}}/dt)/c_0^M$. We have

$$
\begin{aligned}
\delta C \quad = \quad & \left(1 + \sqrt{\frac{v}{v_0}}\right) \mathrm{e}^{-\sqrt{v/v_0}} \, \dot{I}(t) \\
- \quad & \left(1 + \sqrt{\frac{v}{v_0}}\right) \mathrm{e}^{-\sqrt{v/v_0}} \, I(t) \, \frac{\dot{v}}{2\sqrt{vv_0}} \\
+ \quad & \mathrm{e}^{-\sqrt{v/v_0}} \, I(t) \, \frac{\dot{v}}{2\sqrt{vv_0}} \, .
\end{aligned}
\tag{10}
$$

The turning point, $\delta C = 0$, is given by

$$
\frac{\dot{v}}{v_0} = 2\left(1 + \sqrt{\frac{v}{v_0}}\right) \frac{\dot{I}}{I} \, ,
\tag{11}
$$

where $\dot{I}/I$ is the (relative) increase of the incidence. Given that v_0 is about one percent, one arrives hence to the following rule of thumb:

> "For every proportional increase $\dot{I}/I$ of the incidence, one needs to vaccinate an additional percentage of at least twice that amount in order to outrun the virus."

This lower bound holds for $v \to 0$, becoming larger when vaccination progresses. Vaccinating faster reduces daily Covid-19 fatalities, which increase when vaccination speeds falls below the above threshold. We have so far concentrated exclusively on the fact that vaccines are highly effective against severe cases and death, not only in clinical trials, but also in reality, up to 99% among the over 60 [14].

The evidence regarding the impact of vaccines on the spread of infections is less clear [16]. However, any impact of vaccination on infectiousness would not change the balancing condition (11), which applies in general. However, with vaccination reducing infectiousness, the growth of the disease slows down (to a lower value for $\dot{I}/I$), making it easier to reach the point where the curve 'flattens'.

3.4 Time evolution

The balancing condition (10) can be used to analyze a range in different situations. Of interest is to combine (10) with the observed linear increase in vaccination rates, using $v = (t/t_0)^2/2$. One obtains

$$
\frac{t}{t_0} = \left(2v_0 + \frac{t}{t_0}\sqrt{2v_0}\right) t_0 \frac{\dot{I}}{I} \, .
\tag{12}
$$

An important result is that the duration t_0 of the vaccination campaign matters in absolute numbers, given that t_0 rescales $\dot{I}/I$ on the right-hand side of (12). Slow campaigns are substantially less effective in controlling third-wave outbreaks, as measured by the relative increase $\dot{I}/I$ in infected.

The threshold relation (12), which determines whether medical costs are kept from rising, can be rewritten in terms of the two structural parameters p and a_0,

$$\frac{t}{t_0} = \left(a_0^2 p + \frac{t}{t_0} a_0 \sqrt{p} \right) t_0 \frac{\dot{I}}{I}, \qquad v_0 = \frac{a_0^2 p}{2}, \tag{13}$$

see (9).[2]

The parameter a_0 represents a fixed characteristics of the disease. However, the slope p of the population pyramid differs in part substantially across countries, with younger countries having a smaller slope parameter (in absolute terms). Among EU member states, the differences in the slope of the pyramid at old age is minor, as shown in Fig. 1. However some developing countries (e.g. Nigeria) have a much younger population, with pyramid slope parameters about one half of that of Italy. This implies that younger countries can contain health costs even with substantially slower vaccination campaigns. [3]

4   Containment scenarios

For two scenarios, namely for comparatively slow/fast vaccination campaigns, we examine to which extend progress in vaccinating the population can offset the growth of the pathogen.

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[2]We concentrate on the general analytical solution in order to avoid having to make too many specific assumptions about the way the pandemic spreads. For a more detailed, structural approach see [17].

[3]However, countries with a younger population, like Nigeria, are often also much poorer and might face logistical difficulties distributing vaccines which have to be kept constantly at very low temperatures.

Table 1: **Containment thresholds.** For two different vaccination campaign lengths t_0 (in weeks), the maximal weekly increases in infected, $\Delta I = [\dot{I}/I]_{\text{week}}$ that can be contained when vaccinating top-down a progressively larger fraction $v = (t/t_0)^2/2$ of the population. Given are the thresholds for ΔI at distinct stages of the campaign, viz for $t/t_0 = 1/10, 1/4, 1/2, 1$. Note that the absolute value of t_0 matters, as expressed by (12).

t/t_0		1	0.5	0.25	0.1
v in percent		50	12.5	3.2	0.5
EU	$t_0 = 32$	0.19	0.16	0.13	0.09
ISR	$t_0 = 10$	0.59	0.52	0.42	0.27

4.1 Aggressive new mutations can be controlled by fast vaccination campaigns

We denote vaccination campaigns to be fast when it takes about 10 weeks, as for Israel, to fully vaccinate one half of the population. In units of weeks, the balancing condition (12) takes then the form

$$\frac{t}{t_0} = \left(0.22 + 1.48\frac{t}{t_0}\right)\left[\frac{\dot{I}}{I}\right]_{\text{week}}, \tag{14}$$

when using $v_0 \approx 0.011$. An increase in the number of infected, by a weekly rate of $\Delta I = [\dot{I}/I]_{\text{week}}$, will not lead to an increase of the weekly death count if the balancing condition (14) is satisfied. At $t = t_0$, when half of the population has been vaccinated, this can be achieved for $\Delta I = 1/(0.22 + 1.48) = 0.59$, earlier on for respectively smaller ΔI. Fast vaccination campaigns are hence able to control even rapidly expanding new mutations. For a refinement of the above estimates one could use the v_0 appropriate for the country in question, here Israel, which would be slightly smaller. In Table 1 the containment efficiency is listed for selected values of t/t_0.

4.2 Control deteriorates for slow vaccination campaigns

As a second example we consider a comparatively slow vaccination campaign, as for the EU, with t_0 being about 32 weeks. The balancing condition is then

$$\frac{t}{t_0} = \left(0.7 + 4.7\frac{t}{t_0}\right)\left[\frac{\dot{I}}{I}\right]_{\text{week}}. \tag{15}$$

After four months, half way through the campaign, when $t/t_0 = 1/2$, weekly $\Delta I = 0.5/(0.7 + 4.7 * 0.5) = 0.16$ can be controlled.

In reality vaccination campaigns do not strictly follow age, prioritizing also additional groups in occupations requiring close physical contact, like teachers and medical personal. However, it seems that the priority given to these other groups does not have a major impact on the distribution of vaccines. In Israel, one of the few countries with data on vaccination by age, in early January 2021, over 75 percent of the over 60 years old had received at least one dose, but only 15 percent of those between 15 and 59 [18].

4.3 Taking into account the impact of infections on transmission rates

We have so far concentrated on the relationship between the observed value of the increases in cases, ΔI, and the speed of the vaccination campaign needed to prevent an increase in the medical costs. However, one could also consider that progressing vaccination reduces the pool of susceptibles. To illustrate the potential impact of this effect we assume that vaccination fully protects also

against transmission. This might not correspond fully to reality but it has the advantage of providing an upper bound for the benefits from vaccination. The resulting new thresholds are as shown in Table 2.

Public discussion usually concentrates on the need to keep the reproduction number, R, below one, which is equivalent to keeping the growth rate, g, below zero. This goal is easier to achieve when a certain proportion of the population is fully vaccinated and hence immune. The first row in Table 2 indicates that for the EU the vaccination campaign would on its own be sufficient to offset the natural growth in cases with reproduction number, R, up to 1.37 only when 50% of the population has been vaccinated.

The reproduction number of the new variant B.1.1.7 has been estimated to be 0.6 points higher than the original SARS Coronavirus 2 [3]. This implies that without NPIs the growth rate of the new strain is likely to be higher than the threshold that can be reached with the slow EU vaccination campaigns. By contrast, the high speed campaign of Israel could deal with an R value per week of 1.59 already mid-way in the (shorter) vaccination campaign.

More in general, a comparison of Tables (1) and (2) shows that the impact of vaccination on the balancing condition becomes significant only after the second half of the vaccination campaign (i.e. for $t/t_0 \in [0.5, 1]$).

5 Inter-temporal considerations

We have so far concentrated on the evolution of fatalities as a function of the speed of the vaccination campaign and on the combination of the age specific infection fatality rates with the slope of the age pyramid. In principle, policy makers should look at the sum of all future health costs. We provide an analytical solution for this sum below. However, policy making seems to be guided de facto mainly by short-term considerations [19], namely, whether the pressure on the health system and the number of fatalities can be contained in the short run [20].

Table 2: **Containment thresholds with vaccination reducing transmission.** As for Table 1, the maximal weekly reproduction number Δ_I that can be contained, adjusted for the reduction in infections due to vaccination. Given are the thresholds in terms for $\Delta I/(1-v)$ at distinct stages of the campaign, viz for $t/t_0 = 1/10,\ 1/4,\ 1/2,\ 1$. Compare (12).

t/t_0		1	0.5	0.25	0.1
v in percent		50	12.5	3.2	0.005
EU	$t_0 = 32$	0.37	0.19	0.14	0.09
ISR	$t_0 = 10$	1.16	0.59	0.43	0.27

5.1 Health costs saved

The reduction in the risk to die once infected, as given by (4), can be calculated from the product of the percentage vaccinated, v, the age specific case fatality rate and the infection rate. The overall reduction in health costs is then proportional to $R_v v$,

$$
\begin{aligned}
C_{\text{saved}}^{\text{medical}} &= \int_0^{T_v} v R_v \left(I_0 e^{gt} \right) dt \\
&= p I_0 R_{\max} \int_0^{T_v} \left[1 - (1 + g_v t) e^{-g_v t} \right] e^{gt} dt \,,
\end{aligned}
\tag{16}
$$

where T_v is the vaccination period considered. The growth rate g of infections, which is taken here to be given, is related by $g = (R-1)/\tau$ to the reproduction number R, where τ is the characteristic transmission or 'generation' time. It determines the percentage increase of the $\dot{I}/I$, or $I = I_0 \exp(gt)$, where I_0 is the incidence at the start of the vaccination campaign. The integral on the right-hand side of (16) is[4]

$$
\frac{e^{gT_v} - 1}{g} - \frac{e^{(g-g_v)T_v} - 1}{g - g_v} - g_v \int_0^{T_v} t e^{(g-g_v)t} dt \,,
\tag{17}
$$

with

$$
\int_0^{T_v} t e^{(g-g_v)t} dt = \frac{1}{(g-g_v)^2} \left[((g-g_v)t - 1) e^{(g-g_v)t} \right]_{t=0}^{t=T_v} \,.
\tag{18}
$$

Considering (17) and (18) together one finds that the solution contains two parts, one of which depends only on g and the other one depends only on $g - g_v$. A new strand of the virus with a higher infectiousness (i.e. a higher g) would thus require a faster vaccination campaign as already found above.

5.2 Short- and long-term health costs savings

The small-t expansion of (16) is

$$
C_{\text{saved}}^{\text{medical}} \approx p I_0 R_{\max} \int_0^{T_v} \left[\frac{(g_v t)^2}{2} \right] e^{gt} dt \approx \frac{p I_0 g_v^2 R_{\max}}{6} T_v^3 \,.
\tag{19}
$$

Initially, when vaccination rates increase linearly with time, the reduction in health costs achieved by immunizing a growing fraction v of elderly scales only with the third power of the vaccination period T_v. This holds even when the pathogen continues to spread exponentially, which enter only in higher orders.

In the opposite limit, when T_v is large, there is only a finite number of lives to be saved. This is due to the exponential decay of IFR, with decreasing age cohorts. We therefore find

$$
C_{\text{saved}}^{\text{medical}} \approx \frac{p k I_0 R_{\max}}{g} e^{gT_v}
\tag{20}
$$

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[4]We neglect discounting given that the period considered is in weeks. Conventional social discount rates would be essentially be zero of this unit [19].

for large vaccination periods T_v. This relation is however only valid as long as the incidence rate $I = I_0 \exp(gt)$ remains below unity.

Discussion

A key aim for policy makers grappling with a potential third wave of the Covid-19 pandemic in early 2021 is to 'flatten the curve', i.e. to keep hospitalizations and fatalities from rising exponentially [21].

We have concentrated on three key factors shaping the problem. First, the mortality risk from a Covid-19 infection increases exponentially with age. Second, the sizes of age cohorts decrease at the top of the population pyramid. Third, vaccination proceeds at an increasing speed. Taking these factors into account, we derived an expression for a critical threshold, finding that vaccination campaigns above/below this threshold are able/unable to maintain current fatality levels when the daily case numbers continue to raise at a given rate. Countries with a comparatively young/old population have it easier/harder to reach this threshold, which is otherwise directly controlled by the speed of the vaccination program.

Vaccine hesitancy and other factors, such as waning immunity with age, can reduce the overall effect of vaccination campaigns. Moreover, there are other, less age specific cost of the disease, like 'long Covid' [22]. Incorporating these factors would refine the model, making it at the same time necessary to estimate larger numbers of parameters.

A central insight of our study is that it hurts twice when vaccination is slow. On top of the slow increase in the number of the protected, there is a second effect in addition to the direct time delay. Consider two vaccination campaigns, one taking twice the time then the other, $2t_0$ instead of t_0, to vaccinate 50% of the population. The control capabilities halfway through the campaign, respectively at $t = t_0$ and at $t = t_0/2$, are not identical. We find that the capability of slow vaccination programs to control aggressive new Covid-19 strains are not just delayed, but strongly reduced.

Vaccination reduce also the spread of the virus. This provides an additional element which increases the importance of vaccination speed. However, this element becomes significant only in the second half of a vaccination campaign. Here again the length of the vaccination campaign matters as this phase of reduced transmission is reached twice as fast. To vaccinate fast is substantially more important than generally acknowledged.

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