# Modelling the filtration efficiency of a woven fabric: The role of multiple lengthscales

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During the COVID-19 pandemic, many millions have worn masks made of woven fabric, to reduce the risk of transmission of COVID-19. Masks are essentially air filters worn on the face, that should filter out as many of the dangerous particles as possible. Here the dangerous particles are the droplets containing virus that are exhaled by an infected person. Woven fabric is unlike the material used in standard air filters. Woven fabric consists of fibres twisted together into yarns that are then woven into fabric. There are therefore two lengthscales: the diameters of: (i) the fibre and (ii) the yarn. Standard air filters have only (i). To understand how woven fabrics filter, we have used confocal microscopy to take three dimensional images of woven fabric. We then used the image to perform Lattice Boltzmann simulations of the air flow through fabric. With this flow field we calculated the filtration efficiency for particles around a micrometre in diameter. We find that for particles in this size range, filtration efficiency is low ( $\sim 10\%$ ) but increases with increasing particle size. These efficiencies are comparable to measurements made for fabrics. The low efficiency is due to most of the air flow being channeled through relatively large (tens of micrometres across) inter-yarn pores. So we conclude that our sampled fabric is expected to filter poorly due to the hierarchical structure of woven fabrics.

### I. INTRODUCTION

During the COVID-19 pandemic, billions of people have worn masks (face coverings) to protect both themselves and others from infection<sup>1–4</sup>. There are three basic types of mask or face covering. Surgical masks and respirators are made of non-woven materials, while cloth masks are made of woven material. Filtration of air by non-woven materials is well studied<sup>5</sup>. However, pre-pandemic, very little research was done into filtration by woven materials, which have a different structure to that of non-woven materials. Here we try and address this, by studying how a woven fabric filters small particles out of the air.

Woven fabrics have a very different structure from surgical masks, see Fig. 1. Surgical masks are meshes of long thin fibres<sup>5</sup>, of order ten micrometres thick. While fabrics are woven from cotton (or polyester, silk, ...) yarn. Cotton yarn is a few hundred micrometres thick, and is composed of cotton fibres each of order ten micrometres thick. These fibres are twisted into yarns, which are in turn woven into the fabric<sup>6</sup>, see Fig. 1. This two-lengthscale (fibre and yarn) hierarchical structure of fabric is known to affect the fluid flow through fabric, because it has been studied in the context of laundry<sup>7,8</sup>. However, there has been little effort to study its effect in the context of particle filtration<sup>9</sup>.

To understand how woven fabrics filter air, we started by using a confocal microscope to obtain a three-dimensional image of a sample of fabric, at a spatial sampling rate of  $1.8\,\mu m$ . This image is then used as input to Lattice Boltzmann simulations of air flow inside a woven face mask during breathing. That flow field is then used to calculate large numbers of particle trajectories through the fabric to estimate filtration efficiencies.

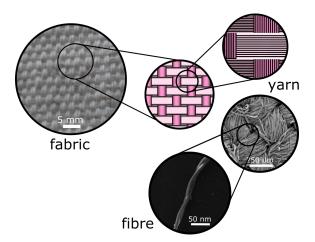


FIG. 1. Fabric is a porous material with structure on multiple length-scales. From left to right we look at successively smaller length-scales. At the largest lengthscale, fabric is a lattice woven from perpendicular yarns that go over and under other yarns at right angles to them. In the left-hand schematic, vertical yarns are shown as dark pink, horizontal yarns as pale pink. As illustrated in both the middle schematic and the SEM images on the right, these yarns are made by twisting together many much smaller fibres. At the right we show a single fibre. Fibres are of order  $10\,\mu m$  across while yarns are a few hundred  $\mu m$  across.

### A. Previous work on filtration by woven fabrics

Konda *et al.*<sup>10,11</sup>, Duncan *et al.*<sup>12</sup> and Sankhyan *et al.*<sup>13</sup>, have all measured filtration efficiencies for a number of fabrics. They all studied the filtration of particles in the size range we consider, which is  $\geq 1 \, \mu m$ . Zangmeister et al.<sup>14</sup> has studied

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filtration for smaller particles. Note that the original measurements of Konda and coworkers suffered from methodological problems <sup>11,15–17</sup>, which were later corrected <sup>11</sup>.

# B. Evidence that droplets approximately a micrometre in diameter carry infectious SARS-CoV-2 virus

The literature on COVID-19 transmission is large but it is worthwhile briefly summarising the part most relevant to this work. The breath we exhale is an aerosol of small mucus droplets in air that is warm and humid because it has come from our lungs  $^{18}$ . These droplets range in size from much less than a micrometre to hundreds of micrometres  $^{19}$ . Vocalisation (i.e. speech or singing) produces more aerosol than ordinary breathing  $^{19-21}$ . The peak in the size distribution function of exhaled droplets is around  $1.6\,\mu\mathrm{m}$  — this is the count median diameter of Johnson and coworkers  $^{19}$ .

The median diameter of  $1.6\,\mu m$  is for droplets as exhaled in our breath, which is essentially saturated with water vapour. When our breath mixes with room air, the humidity drops and droplets of this size evaporate in much less than a second<sup>22</sup>. After this evaporation, the droplet approximately halves in diameter<sup>19,22</sup>. So, typical droplet sizes are around  $1.6\,\mu m$  as we breath them out through a mask, but around  $0.8\,\mu m$  when we breath them in. These are approximate medians of broad distributions.

Coleman and coworkers<sup>23</sup> found SARS-CoV-2 viral RNA in both particles with diameters smaller than and larger than  $5\,\mu m$ , and found that most of the viral RNA was in droplets with diameters less than  $5\,\mu m$ . These diameters are after evaporation. Santarpia and coworkers<sup>24</sup> found infectious virus in particles both with diameters  $< 1\,\mu m$  and in the range 1 to  $4\,\mu m$ , but not in particles larger than  $4.1\,\mu m$ . Hawks and coworkers<sup>25</sup> were also able to obtain infectious virus in aerosols smaller than  $8\,\mu m$ . It should be noted that the study of Hawks and coworkers was of infected hamsters not humans. Finally, Dabisch and coworkers infected macaques with an aerosol of droplets with median diameter  $1.4\,\mu m^{26}$ . This body of very recent work suggests that aerosol particles of order a micrometre carry most of the virus.

It is also worth noting that Coleman and coworkers<sup>23</sup> also found that amount of viral RNA varied widely from one person to another. Some infected people breathed out no measurable RNA. Those that did breathed out an amount that varied by a factor of almost a hundred. Viral RNA was found even for those who never developed COVID-19 symptoms, i.e., who always remained asymptomatic.

As we state above, we use 'droplet' to cover all sizes from much less than a micrometre to hundreds of micrometres and more. This is in line with the aerosol and fluid mechanics literature, but some work in the medical literature reserves 'droplet' for diameters over  $5 \,\mu m$ , despite there being no justification for this distinction<sup>27,28</sup>.

#### C. Evidence that masks filter out SARS-CoV-2

Adenaiye and coworkers<sup>29</sup> studied the effect of masks on the amount of viral SARS-CoV-2 RNA breathed out. This study tested a wide range of masks as participants were asked to bring their own masks. They found that in 'fine aerosols ( $< 5\mu m$ )', the masks reduced the amount of viral RNA detected by 48% (95% confidence interval 3 to 72%), while for larger aerosols, masks reduced the viral RNA by 77% (95% confidence interval 51 to 89%). Here,  $5\mu m$  is presumably the evaporated diameter (not radius) but this was not specified by the authors.

#### D. Mechanism of filtration

Filtration is traditionally ascribed to a sum of four mechanisms<sup>5</sup>. The idea being that a particle with zero size, zero inertia, zero diffusion, and zero charge, will follow the streamlines perfectly and not be filtered out. However, deviations from any one of those four conditions can cause a collision and hence filtration.

The four mechanisms are:

- 1. *Interception:* Particles whose centre of mass follows streamlines perfectly can still collide with fibres, if the particles have a finite size. This is a purely geometric mechanism, that does not require inertia.
- 2. *Inertial:* With inertia, particles cannot follow the air streamlines perfectly. Whereas a streamline goes around an obstacle, a particle with inertia will deviate from the streamline and so may collide.
- Diffusion: Particles diffuse in air, creating further deviations from streamlines and thus potential collisions with the obstacle.
- 4. *Electrostatic interactions:* Charges, dipole moments etc, on the fibres and on the droplets will interact with each other. If they pull the two towards each other, this will enhance filtration. Cotton fibres have no charge distribution as far as we know, so we do not expect this to be a significant mechanism here.

Note that in practice these mechanisms are never completely independent<sup>5</sup>.

Flow through masks is sufficiently slow, and the length-scales are sufficiently small, that the flow is close to Stokes flow, i.e., the Reynolds number is small. This means that streamlines do not depend on flow speed/pressure difference. In turn, this implies that interception filtration is independent of flow speed. Inertial filtration becomes more important with increasing flow speed, as the faster moving particles have more inertia. While diffusion filtration becomes less efficient at faster flow speeds, as then particles spend shorter times passing through the mask. The particles then have less time to diffuse into the material of the mask, and be filtered out.

Here, we will focus on particles a micrometre and larger, where diffusion is less important as a filtration mechanism because particles this large diffuse slowly. So we will focus on interception and inertial filtration. However, in the conclusion we will return to filtration by diffusion and argue that filtration by diffusion in our fabric should be very inefficient.

The remainder of this paper is laid out as follows. The next section describes how we imaged the fabric and analysed the imaging data. The third section describes our Lattice Boltzmann (LB) simulations of air flow through the mask. Then the next section characterises this air flow. The fifth and sixth sections have our method for calculating particle trajectories and our results for filtration, respectively. The seventh section briefly discusses filtration via diffusion. The last section is a conclusion.

# II. IMAGE ACQUISITION AND ANALYSIS OF A SAMPLE OF WOVEN FABRIC

The fabric was obtained from a commercial fabric mask. Square pieces of 1, 2.25 and  $4\,\mathrm{cm^2}$  were weighed individually, giving a mass per unit area of  $120\,\mathrm{g\,m^{-2}}$ , see Table I. Using brightfield optical microscopy (Leica DMI3000 B) with a Leica 4x objective, we estimated the thickness of the fabric in air to be  $285\pm24\,\mu\mathrm{m}$ , which we determined through different measurements along the fabric. Using the mass density of cotton,  $\rho_c$ , from Table III, this corresponds to the fabric being on average about 28% cotton fibres and 72% air.

# A. Image acquisition

In order to study the 3D structure of the fabric, square pieces of 0.5 cm of cotton were dyed with fluorescein (Sigma Aldrich) following Baatout *et al.*<sup>30</sup>. The dyed cotton squares were then washed in deionised water to eliminate any dye excess and left to dry under ambient conditions for 48 hours. Once dried, the fabric was re-submerged in 1,2,3,4-tetrahydronaphthalene (tetralin, Sigma Aldrich). We chose this solvent due to its refractive index being close to the index of cotton ( $\eta_{D\text{tetralin}} = 1.544^{31}$  and  $\eta_{D\text{cotton}} = 1.56 - 1.59^{32}$ ). Such matching is needed to allow imaging with fluorescence confocal microscopy.

The dyed fabric samples were immersed in tetralin. They were confined in cells constructed using three coverslips on a microscope slide. Two of the coverslips acted as a spacer, and they were sealed using epoxy glue. The spacing coverslips

Area of sample (cm <sup>2</sup> )	mass (g)	mass/area (g cm <sup>-2</sup> )
1	0.01210	0.01210
2.25	0.02742	0.01219
4	0.04813	0.01203

TABLE I. Table of measurements of the mass of samples of the fabric, used to determine its mass per unit area.

have a height of 0.56 mm, which prevented fabric compression. A confocal laser scanning microscope Leica TCS SP8 equipped with a white light laser, was used to study the fibre structures, using a Leica HC PL APO 20x glycerol immersion objective with a 0.75 numerical aperture and a correction ring. The excitation/emission settings used for the fluorescein dye were 488 and 500 nm, respectively. Scans of the cell in the z-axis were acquired to analyse the fibre network in 3D, where care was taken to ensure the pixel size (1.8  $\mu$ m) was equal along all axes.

The confocal microscopy data is in the form of a stack of  $n_z = 62$  images of the xy plane, each of which is  $n_x = 756$  by  $n_y = 756$  voxels. Each voxel is a cube of side 1.8 µm, see Table II. Slice number 19 (starting at zero) is shown in Fig. 2. In each slice, approximately two-thirds of the field of view is taken up with a strip of the fabric, which runs left to right in Fig. 2.

Of the 62 slices, image quality in the bottom ten is poor, due to attenuation from the imperfect refractive index matching. So in effect, we can obtain good images for 52 slices, i.e. we can reliably image a section of fabric that is approximately  $93.6\,\mu m$  thick.



FIG. 2. Slice (number 19, starting at 0) of the confocal image of the fabric. Slice is in the *xy* plane. The area simulated using LB is enclosed by a white box.

# B. Fibre size distribution

To obtain estimates of the distribution of fibre diameters we imaged the surface of the fabric using a scanning electron microscope (FEI Quanta 200 FEGSEM, Thermo Fisher Scientific), see Fig. 3. We then estimated the diameter of at least 50 fibres from this image, and obtained the mean and standard deviation of fibre diameters as  $16.7 \pm 4.8 \,\mu\text{m}$ ,which we determined by analysing SEM images.

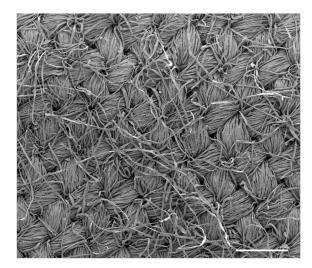


FIG. 3. A Scanning Electron Microscope (SEM) image of the surface of our fabric. The fabric has been coated with gold/palladium. Secondary electron images were taken at 8 kV with a 100x magnification. Scale bar =  $500 \, \mu m$ .

### C. Image analysis

The analysis of the image stack output by the confocal microscope was performed in Python using the the OpenCV<sup>33</sup> and cc3d<sup>34</sup> packages. The confocal image stack is processed as follows:

- 1. We first delete the fibre voxels in the bottom ten slices due to the poorer image quality, leaving us with 52 slices of imaged fabric. We then add 200 slices to the top, and 200 slices to the bottom, all of each are of entirely zero intensity voxels. These additional slices are needed as the array produced for the simulations needs to cover fluid flow into and out of the fabric, i.e., we cannot just simulate flow inside the fabric, we need the approach and exit flows.
- 2. We then blur the image by convolving with a three-dimensional Gaussian filter that is implemented as a sequence of 1-D convolution filters, with a standard deviation  $\sigma_B = 1$  voxel side (1.8 µm).
- 3. Next we threshold the blurred image, setting all voxels with values less than the threshold value T=10 to zero, and all voxels greater than or equal to the threshold value to one. Thus we get a binary image.
- 4. Then we use a 3D connected components algorithm to identify the connectivity of voxels that are one. We assign each voxel with value one to a cluster of connected voxels. All voxels of value one that are part of clusters of size  $N_{CL} = 25$  or less are set to zero, all other voxels of value one, are assumed to be fibre voxels. N.B. Applying the Gaussian filter greatly reduces the number of connected clusters we obtain.

It is worth noting that step four only deletes a total of 507 voxels while keeping 11681929 voxels so deleting a few isolated clusters has very little effect, and that in the final array almost 99.9% of the voxels are part of the largest cluster. So the remaining clusters are very small. We should expect most voxels to be in a single cluster, as the fabric should connected in order not to fall apart<sup>6</sup>.

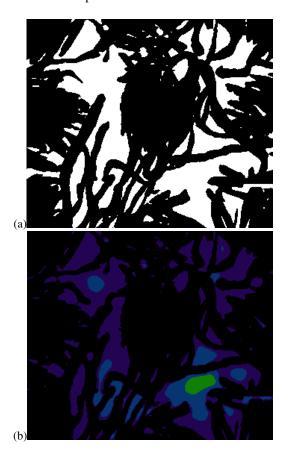


FIG. 4. (a) The thresholded and so binary image produced by image analysis of the area in the white box in Fig. 2. Fibre voxels are in black and air voxels are in white. (c) Heatmap of the z component of the velocity in the same area. Again black is the fabric. Dark purple, blue and pale green are velocities less than the mean, between the mean and ten times the mean, and over ten times the mean velocity, respectively. Both images are  $594\,\mu\text{m} \times 504\,\mu\text{m}$ .

### D. Region of the fabric studied

The fabric is essentially a rectangular lattice, woven from yarns that cross at right angles. Estimated lattice constants are in Table II. The lattice constants are around 20 times the average fibre diameter.

We want to model a representative part of the fabric of a face covering, so we study an area of two by two lattice sites. This area is shown by the white box in Fig. 2, and in Fig. 4(a). Note that we put the edges of the white rectangle in the densest part of the fabric where flow is least. The dimensions of the

Quantity	Value			
Fabric imaged				
cubic voxel side length	1.8 µm			
total thickness imaged	$62 \text{ voxels} = 111.6 \mu\text{m}$			
thickness used	$L_F = 52 \text{ voxels} = 93.6 \mu\text{m}$			
area imaged	$756 \times 756$ voxels			
	$= 1360.8  \mu \text{m} \times 1360.8  \mu \text{m}$			
area used	$n_x = 310 \text{ to } 310 + 330$			
	$n_y = 280 \text{ to } 280 + 280$			
	$= 594  \mu \text{m} \times 504  \mu \text{m}$			
yarn lattice constants	$297\mu m$ and $252\mu m$			
Threads per inch (TPI)	186			
Lattice Boltzmann parameters				
box size $n_x \times n_y \times n_z$	$330 \times 280 \times 462$			
•	$= 594 \mu\text{m} \times 504 \mu\text{m} \times 471.6 \mu\text{m}$			
Darcy velocity $U = Q/A$	$5.6 \times 10^{-7}$			
Re for lengthscale 297 µm	$6 \times 10^{-4}$			
pressure drop	$6.7 \times 10^{-6}$			

TABLE II. Table of parameter values for the fabric we have imaged, and for our Lattice Boltzmann simulations. TPI is calculated by adding together number of yarns per inch along x and along y.



FIG. 5. Snapshot of the movie in supporting information that shows the part of the fabric we calculate the flow field for. Rendering done using Blender<sup>35</sup>. (Multimedia view)

white rectangle are given in Table II. A full three-dimensional rendering of the region we study is shown in the Supporting Information, with a snapshot in Fig. 5.

# E. Estimation of what fraction of the fabric thickness is in our simulation box

Using a mass density for cotton in Table III, then simply counting each voxel as  $(1.8\,\mu\text{m})^3$  of cotton, we have a mass/unit area of cotton of  $96\,g\,m^{-2}$  in our fabric array of  $330\times280\times52$  voxels. Our directly measured value is  $120\,g\,m^{-2}$ , so this estimate is that our 52 slices or 93.6  $\mu$ m of fabric contains 80% of the mass of the fabric. However, our estimate for the fabric thickness using optical microscopy is  $285\,\mu\text{m}$ , three times the thickness of our image.

The thickness of fabric measured in air is not perfectly well defined, the fabric is compressible being mostly air and at the

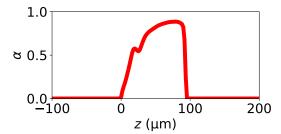


FIG. 6. Plot of the fraction of voxels belonging to a fibre  $\alpha$  (averaged over x and y), as a function of z. The zero of z is at the top of the fabric (slice 0). This is for the volume used in our simulations.

edges there are stray fibres. We have plotted the average fraction  $\alpha$  of voxels that are fibre voxels, as a function of z in Fig. 6. Note that this is measured in solvent. It is mostly above the average value of 28% we obtained in air, and the average value  $\alpha$  inside the fabric of this plot is 69%. It is possible that the fabric may have compacted and/or the fibres swollen in our solvent.

To conclude, there is significant uncertainty in what fraction the fabric thickness is included in the 52 slices. We can only say that our 52 slices contains at least one third of the fabric, but probably no more than two-thirds.

# III. LATTICE BOLTZMANN SIMULATIONS OF AIR FLOW THROUGH FABRIC

Lattice Boltzmann (LB) simulations are performed on a three-dimensional lattice of  $n_x$  by  $n_y$  by  $n_z$  lattice sites; z is the flow direction. Our code is the Palabos LB code from the University of Geneva<sup>36</sup>. The code uses a standard one-relaxation-time LB algorithm on a cubic D3Q19 lattice. The speed of sound  $c_s = 1/\sqrt{3}$  in LB units where both the lattice spacing and the time step are set to one<sup>37</sup>. It has a kinematic viscosity  $v_{\text{LB}} = c_s^2 \left( \omega^{-1} - 1/2 \right)$ . We set the relaxation rate  $\omega = 1$  in LB units, giving a kinematic viscosity  $v_{\text{LB}} = 1/6$  in LB units<sup>37,38</sup>.

We run the LB simulations until the change in mean flow speed along z is very small so we are at steady-state. We then insert particles into the resulting steady flow field to evaluate their trajectories.

Our code reads in the  $330 \times 280 \times 462$  array obtained from our image analysis. Fibre voxels have standard LB on-site bounce back<sup>39,40</sup> to model stick boundary conditions. During each timestep the velocities at all fibre sites are therefore reversed.

The box is configured such that the *x* and *y* edges are in denser parts of the fabric so there is little flow near and at these edges. In the LB simulations we use periodic boundary conditions (PBCs) along the *x* and *y* directions. The real fabric is not perfectly periodic and so our flow field has artifacts near the edges. However, there is no way of avoiding artifacts at the edges, and PBCs are a simple choice.

We impose a pressure gradient along the z axis, to drive flow. We do this by fixing the densities in the first and last xy slices of the lattice along z. We fix the density in the z = 0 slice to be  $1 + 10^{-5}$ , and that in the  $z = n_z - 1$  slice to be  $1 - 10^{-5}$ . This corresponds to a pressure difference of  $(2/3) \times 10^{-5}$  across the fabric.

This small density/pressure difference across the fabric is chosen to keep the Reynolds number small, so we have Stokes flow. The Reynolds number for flow with characteristic lengthscale L is

$$Re = \frac{UL}{v} \tag{1}$$

for v the kinematic viscosity and U the velocity. For the velocity we use the Darcy velocity, see section IV A. The Reynolds number for the largest lengthscale (yarn lattice constant along x) in our simulation box is in Table II and is much less than one so we have Stokes flow in our simulations.

For an air flow speed of  $2.7\,\mathrm{cm\,s^{-1}}$  (moderate exercise) the Reynolds number for air flow with a characteristic lengthscale of a few hundred micrometres is Re  $\simeq 1$ . So in a fabric mask there will small deviations from Stokes flow, but we expect them to have little effect.

The LB simulations only give a flow field on a cubic lattice, so we use trilinear interpolation to give a continuous flow field  $\vec{u}(\vec{r})$ . Trilinear interpolation is the extension to three dimensions of linear interpolation in one dimension<sup>41</sup>.

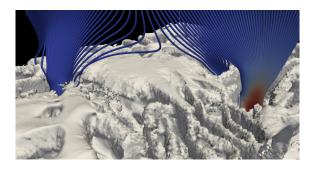


FIG. 7. Plot of the fabric surface (white) together with streamlines. The streamlines are colour coded with local velocity: blue is slow, red is fast. The flat region in the centre of the image is the top of a yarn. Image produced by ParaView<sup>42</sup>.

# IV. AIR FLOW THROUGH THE WOVEN FABRIC

The air flow through the fabric is heavily concentrated in the inter-yarn pores, and there is essentially no flow through the centres of the yarns. This can be seen in the heatmap of the z velocity in Fig. 4(b). Note that all the fastest voxels (shown in pale green) are in a single patch in the middle of the biggest inter-yarn gap. There are 718 of these voxels, out of 27,190 air voxels, and they contribute over a third of the total air flow through this slice.

The flow through the fabric is illustrated by streamlines in Fig. 7. Note that all the streamlines shown flow around

the yarns and through the gaps between the yarns. We conclude that as the air goes through inter-yarn pores, the filtration efficiency will depend on whether or not particles flowing through these pores, collide with the pore sides, or stray fibres across these pores.

The spacing between fibres of a yarn is mostly too small to be resolved by our imaging technique, so presumably is mostly a micrometre or less. Note that the integrity of yarns relies on large numbers of physical contacts  $^6$ , so the fibres must touch in many places. Our limited resolution means we cannot model any flow in between the fibres. However, as the inter-yarn gaps are  $\sim 50\,\mu m$  across, the flow through any gaps between fibres of order  $\sim 1\,\mu m$  or less will be negligible. Poiseuille flow  $^{7.8}$  predicts than any flow in sub-micrometre inter-yarn gaps will be thousands of times slower than flow in the inter-yarn pores.

Finally, the fact that the bottom-right inter-yarn pores has the largest air flow illustrates that the fabric is disordered. It is not a perfect lattice of inter-yarn pores, each of which is the same. This also means that small (in the sense of difficult to detect with the naked eye) amounts of damage to fabric significantly affect flow through it.

### A. Darcy's law

Fluid flow through fabric has been studied in earlier work on the washing of fabric (laundry). Removing dirt from fabric relies on the flow of water through the fabric <sup>7,8,43,44</sup>. These earlier workers, starting with the pioneering work of van den Brekel<sup>7</sup>, assumed that inter-yarn flow was dominant, which is corroborated by the present work. They modelled the flow through fabric using the standard approach for (low Reynolds number) flow through porous media: Darcy's Law.

A mask is a porous medium, and so at low Reynolds number the air flow Q through the fabric is given by Darcy's Law<sup>45</sup>

$$Q = \frac{kA}{\mu} \frac{\Delta p_F}{L_F} \tag{2}$$

which defines the permeability k. Q is the volume of air crossing the fabric per unit time, A is the area of the fabric the air flows through and  $\mu$  is the viscosity of air.

For our thin fabric there are end effects. We neglect these and just consider the pressure drop across the fabric,  $\Delta p_F$  and the thickness of the fabric,  $L_F$ . The flow Q is proportional to the size of the pressure drop across the fabric  $\Delta p$  and inversely proportional to the thickness  $L_F$  of the fabric. The Darcy velocity U is defined by

$$U = \frac{Q}{A} \tag{3}$$

In free space U is the actual flow velocity, while inside a porous medium, some of the area A is occupied by the solid material and so does not contribute to Q. Then the local flow velocity varies from point to point and is mostly higher than the Darcy velocity U.

In our LB simulations we impose the pressure difference  $\Delta p$  (via setting the densities at bottom and top along z), measure Q, and evaluate the permeability from

$$k = \frac{Q\mu}{A} \frac{L_F}{\Delta p_F} \tag{4}$$

The viscosity of our LB fluid is  $\mu = \rho_{LB}v_{LB} = 1/6$ , because  $\rho_{LB} = 1$  is the mass density in LB units and  $v_{LB} = 1/6$  is the kinematic viscosity also in LB units. In the same units  $L_F = 52$ .

We find a permeability of  $k \simeq 0.73$  in LB units, or  $k \simeq 2.4 \,\mu\text{m}^2$  on conversion using our known voxel size. This value is comparable to the value  $k \simeq 4 \,\mu\text{m}^2$  found for cotton sheets (with water as the fluid) in the experiments of van den Brekel<sup>7</sup>.

Note that our fabric is imaged in liquid and van den Brekel's measurements are for fabric immersed in a liquid. So it is possible that in both cases the cotton may have swelled due to absorbing the liquid, reducing k. We imaged the masks in SEM (under vacuum) before and after immersion in tetralin for confocal imaging and observed no change. While of course it is possible that swelling occurred *during* immersion in said solvent we find no evidence for irreversible change due to immersion in tetralin.

### B. Impedance and pressure drop across fabric

The pressure drop across a mask must be low enough to allow easy breathing through the mask. As we have Stokes flow the pressure drop is linearly proportional to the flow velocity, and the proportionality constant defines the mask's impedance  $I^{17}$ 

$$\Delta p_F = IU \tag{5}$$

Using Eq. (2) and Eq. (3), we have

$$I = \mu L_F / k \tag{6}$$

Using the viscosity of air and our estimated k,  $I = 7.1\,\mathrm{Pa\,s\,cm^{-1}}$ . This is the same order as Hancock  $et~al.^{17}$  find for 300 TPI cotton. Konda  $et~al.^{11}$  finds an impedance of  $4.2\,\mathrm{Pa\,s\,cm^{-1}}$  for a 180 TPI cotton/polyester blend. Sankhyan  $et~al.^{13}$  find pressure drops in the range 40 to 55 Pa for a air speed of  $8\,\mathrm{cm\,s^{-1}}$ , which gives impedances in the range 5 to  $7\,\mathrm{Pa\,s\,cm^{-1}}$ . Hancock et al.  $^{17}$  estimate that the American N95 standard for breathability requires a maximum impedance of around  $30\,\mathrm{Pa\,s\,cm^{-1}}$ , four times our fabric's value.

# 1. Model for the Darcy's Law permeability

Van den Brekel<sup>7</sup> uses the Kozeny, or Kozeny-Carman, model for k. This model was developed for beds composed of packed spheres. Although as van den Brekel proposed the vast majority of the flow is through inter-yarn pores, these pores do not resemble the gaps between the sphere in beds of packed spheres. They are channels partially obstructed by stray fibres.

Thus we model k of our fabric by Poiseuille flow in cylinders of effective diameter  $d_{EFF}$  that occupy an area fraction  $\varepsilon_{by}$  of the fabric. This gives

$$k \sim \frac{\varepsilon_{by} d_{EFF}^2}{32} \tag{7}$$

We estimate the effective free diameter to be in between a fibre diameter and a yarn diameter,  $d_{EFF} \sim 50\,\mu\text{m}$ , while the area fraction of inter-yarn pores  $\varepsilon_{by} \sim 0.1$ . These values give  $k \sim 8\,\mu\text{m}^2$  — the same order of magnitude as our measured value. This is consistent with the flow being predominantly through pores tens of micrometres across, that occupy about ten percent of the total area.

#### C. Curvature of streamlines

The inertia of a particle only affects its motion when streamlines are curving. For flow that is just straight ahead the particle will just follow the flow. So we need to characterise the curvature of the streamlines going through the fabric. We do this by determining a characteristic lengthscale for this curvature, which we call  $\Sigma$ .

The lengthscale  $\Sigma$  for curvature of a streamline at a point on a streamline of the flow field is defined by

$$\Sigma = \frac{\vec{u}.\vec{u}}{a_{\perp}} \tag{8}$$

for  $\vec{u}$  the flow field at that point, and  $a_{\perp}$  the magnitude of the normal component of the acceleration  $\vec{a}$  along the streamline at this point. Streamlines are defined by velocities and accelerations and so one way to obtain a lengthscale is the square of a velocity divided by an acceleration.

The acceleration is that along the streamline, i.e., rate of change of streamline velocity while being advected along the streamline. The normal component is obtained by subtracting the parallel component, from  $\vec{a}$ 

$$\vec{a}_{\perp} = a - \hat{u}(\hat{u}.\vec{a}) \tag{9}$$

We have plotted  $\Sigma$  along a set of streamlines in Fig. 8. The local curvature along streamlines within the fabric varies greatly but is mostly around tens to hundreds of micrometres. This is different from the flow in a mesh of single fibres, as found in surgical masks. In surgical masks there is only one lengthscale, that of the fibre diameter, which is typically around  $15\,\mu\text{m}^9$ . So in non-woven filters such as surgical masks, the curvature lengthscale is expected to approach  $15\,\mu\text{m}$  for trajectories near the surfaces of fibres.

# V. CALCULATING PARTICLE TRAJECTORIES AND COLLISIONS

In this section we first introduce the theory for particles moving in a flowing fluid, then describe the details of our calculations.

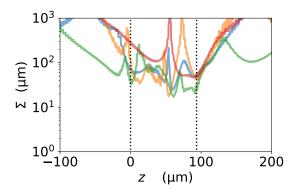


FIG. 8. Plot of the local curvature  $\Sigma$  along four streamlines, as a function of their position along the flow direction z. The vertical dotted lines mark the start and end of the fabric, so outside of these lines we are outside the fabric. N.B. the curves are not smooth because  $\Sigma$  depends on an acceleration. The flow field velocity is obtained by interpolation so the velocity is continuous but its derivative the acceleration is not.

# A. Theory for a particle in a flowing fluid

The particles are spheres of diameter  $d_p$ , that feel only the Stokes drag of the surrounding air. We neglect any perturbation by the particles of the flow field, and assume that the drag force on a particle couples to its centre of mass. Then Newton's Second Law for the particle becomes

$$m_p \frac{d\vec{v}}{dt} = -\frac{3\pi\mu d_p}{C} (\vec{v} - \vec{u}) \tag{10}$$

for a particle of mass  $m_p$  and velocity  $\vec{v}$  in a flow field  $\vec{u}$  of fluid with viscosity  $\mu$ . Here C is the Cunningham slip correction factor <sup>46,47</sup>. We consider particles with  $d_p \ge 1 \, \mu \text{m}$  (due to limited imaging resolution). In this size range C is always close to one (within 15%). Therefore, we just set C = 1 here.

The particles are spheres of mucus which we assume has the mass density of water,  $\rho_p$ . Then  $m_p = (\pi/6)d_p^3\rho_p$ , and Eq. (10) becomes

$$\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = -\frac{18\mu}{\rho_p d_p^2 C} (\vec{v} - \vec{u}) = -\frac{(\vec{v} - \vec{u})}{t_I}$$
(11)

where we have introduced  $t_I = \rho_p d_p^2 C/(18\mu)$ : the timescale for viscous drag to accelerate the particle.

# 1. The Stokes number

The ratio of the timescale  $t_I$  to the timescale for fluid flow to change direction as it goes round an obstacle of size  $L_O$ , defines the Stokes number

$$St(d_p, L_O, U) = \frac{t_I}{L_O/U}$$
 (12)

where we use the Darcy speed U. Then

$$St(d_p, L_O, U) = \frac{\rho_p d_p^2 UC}{18\mu L_O} \sim \frac{3.08 \times 10^6}{\text{m}^2 \text{s}^{-1}} \frac{d_p^2}{L_O} U, \quad (13)$$

Parameter values in table III were used. For  $St \ll 1$  viscous forces dominates inertia and the particle follows streamlines faithfully. However, for  $St \gg 1$ , inertia dominates and the particle's trajectory will strongly deviate from streamlines. As the streamlines go round obstacles, deviating from streamlines can result in the particle colliding with an obstacle and being filtered out. This is inertial filtration.

The Stokes number depends on the flow speed, and on both the size of the particle and of the obstacle the flow is going around. Figure 9 shows the Stokes number as a function of particle diameter, for particles in flows fields curving over lengthscales of 10 and 100  $\mu m$ . Note that for flow fields curving over a distance  $10\,\mu m$ , a Stokes number of one is only reached for particles greater than  $10\,\mu m$  in diameter. So our fabric where the curvature  $\Sigma$  is mainly at least tens of micrometres (see Fig. 8) is expected to show little inertial filtration of any particle around  $10\,\mu m$  or smaller in diameter.

Quantity	Value	Reference		
Air				
mass density	$1.2 \mathrm{kg} \mathrm{m}^{-3}$	48		
dynamic viscosity $\mu$	$1.8 \times 10^{-5}  \text{Pa}  \text{s}$	48		
kinematic viscosity v	$1.5 \times 10^{-5}  \text{m}^2  \text{s}^{-1}$	48		
Water/mucus				
mass density $\rho_p$ (water)	998 kg m <sup>-1</sup>	48		
dynamic viscosity (mucus)	0.1 Pas	49		
mucus/air surface tension $\gamma$	$0.05{\rm Nm^{-1}}$	49		
Cot	ton fibres			
mass density $\rho_c$	$1500  \mathrm{kg}  \mathrm{m}^{-3}$	50		
Typical breathing flow rates				
tidal breathing at rest	$61 \mathrm{min}^{-1}$	51		
during mild exertion	$201 \mathrm{min}^{-1}$	51		
during moderate exertion	$301 \mathrm{min}^{-1}$	51		
during maximal exertion	$851 \mathrm{min}^{-1}$	51		
Average flow speeds				
effective mask area	190 cm <sup>2</sup>	52		
flow speed (rest)	$0.5{\rm cms^{-1}}$			
flow speed (mild)	$1.8{\rm cms^{-1}}$			
flow speed (moderate)	$2.7  \mathrm{cm}  \mathrm{s}^{-1}$			
flow speed (maximal)	$7.5{\rm cms^{-1}}$			

TABLE III. Table of parameter values for masks, air, water and mucus; all at 20 °C and atmospheric pressure 10<sup>5</sup> Pa. Note that small droplets dry rapidly and this will cause their viscosity to increase. Flow rates are determined from the volume typically exhaled during one minute. Moderate exertion is defined as that readily able to be sustained daily during 8 hours of work, whereas maximal exertion is the upper limit of what can be sustained for short periods of time (e.g. during competitive sports). Flow speeds are calculated for the stated mask area and flow rates assuming perfect face seal.

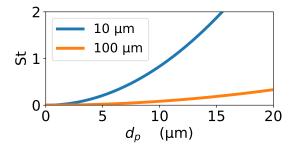


FIG. 9. Plot of the Stokes number as a function of particle diameter  $d_p$ , using Eq. (13). The blue and orange curves are for obstacle sizes  $L_O = 10 \, \mu \text{m}$  and  $100 \, \mu \text{m}$  respectively. The flow speed is set to  $U = 2.7 \, \text{cm s}^{-1}$ .

# B. Evaluation of filtration using our Lattice-Boltzmann flow field

We simulate  $N_{\text{samp}}$  particle trajectories in our flow field, to estimate the filtration efficiency. The filtration efficiency is estimated from the fraction of the flux of particles that collide with the fabric.

Filtration efficiency = 
$$\frac{\sum_{i}^{\text{coll}} v_{zi}}{\sum_{i}^{\text{coll}} v_{zi} + \sum_{i}^{\text{pen}} v_{zi}}$$
(14)

where the sum with superscript 'coll' is over all particles that collided with a fibre voxel, and the sum with superscript 'pen' is over all particles that pass through the fabric without colliding.  $v_{zi}$  is the z component of the velocity of particle i at the starting point of its trajectory. Note that as we are interested in the fraction of the flux filtered, each particle is weighted by the local velocity. We assume the particle concentration is uniform in the air, so regions where the air is flowing faster contribute more to the flux than where the regions are flowing more slowly.

See Appendix A for further details of how we compute trajectories. All calculations are for flow at the speed  $U = 2.7 \,\mathrm{cm}\,\mathrm{s}^{-1}$ , corresponding to breathing under moderate exertion (see Table III).

### VI. RESULTS FOR PARTICLE FILTRATION

In Fig. 10 we have plotted results for the fraction of particles that collide with a fibre and are filtered out, as a function of the diameter of the particle. These are the red data points. We see that the efficiency is less than 10% for micrometre sized particles, and although it increases with increasing size we are still filtering less than half of the particles at a diameter of 10 µm. We breathe out droplets with a wide range of sizes but the peak of this distribution is around one micrometre 19. We predict that the fabric we have imaged is very poor at filtering out droplets of this size. But note that we could only image approximately half of one cotton fabric layer; presumably the filtration efficiency of the full layer is higher.

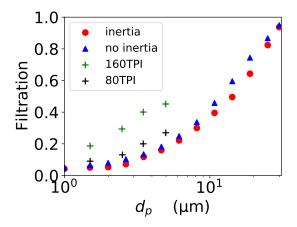


FIG. 10. Plot of the fraction of particles filtered, as a function of their diameter  $d_p$ . This is in air with flow speed  $U=2.7\,\mathrm{cm\,s^{-1}}$ . The blue and red circles are with the inertia of a particle with mass density of water, and without inertia. They are each averages over  $N_{\mathrm{samp}}=1600$  particle trajectories. The green and brown pluses are measurements of Konda et al. (obtained from Fig. 2(B)<sup>53</sup>). These measurements are for a pressure drop across the fabric of 10 Pa, whereas at our value of U, the estimated pressure drop is 19 Pa. The impedances measured by Konda et al. 11 are lower than our value (7.1 Pa s cm<sup>-1</sup>), they find values of 1.3 Pa s cm<sup>-1</sup> for 80 TPI, and 4.2 Pa s cm<sup>-1</sup> for 160 TPI. Thus, especially for the 80 TPI fabric, although their pressure drop is lower, the air speed is higher.

Both Konda *et al.*<sup>10,11</sup> and Duncan *et al.*<sup>12</sup> have measured the filtration efficiency of woven fabrics, for particles up to five micrometres. Both groups find a large variability in filtration efficiency from one material to another, with filtration efficiencies in the range less than 10 to almost 100%, for particles with diameters of a few micrometres. Sankhyan *et al.*<sup>13</sup> found comparable filtration efficiencies to Konda *et al.*and Duncan *et al.*. They also found that the fabric masks were systematically less good at filtering than non-woven surgical masks.

Two data sets from Konda et al. 11 are plotted in Fig. 10. Konda et al. 10,11 found that the filtration efficiency of fabric increased with its TPI. In Fig. 10 we see that they found that the filtration efficiency for 160 TPI cotton/polyester fabric is higher than for 80 TPI cotton. We estimate that our fabric's TPI is 186. Our efficiencies are lower than those measured by Konda et al. 11 but the slope is very similar. At a diameter of 1.5 µm we find an efficiency of 5%, whereas Konda et al. 11 find efficiencies of 9 and 19% for TPIs of 80 and 180, respectively. Our model makes a number of approximations: flow field on a 1.8 µm lattice, coupling at centre of mass, etc, so our estimated efficiencies are likely only accurate to within a factor of two. Thus, within our large uncertainties our results are essentially consistent with the measurements.

# A. Inertia can cause collisions to be avoided and so reduce filtration efficiency

In order to understand the role of inertia in filtration by woven fabric, we calculated the filtration without inertia. The results are shown as blue circles in Fig. 10, and are for pure interception filtration. If we compare those points with the red points, which are with inertia, we see that the difference is small. Inertia has a small effect and filtration is mainly interception.

But the difference is that the effect of inertia is to slightly decrease filtration. We have found that the effect of inertia can be to cause a collision that occurs without inertia to be avoided, see Fig. 10. There we have plotted two trajectories with the same starting point but with inertia (purple) and without inertia (orange). The particle with inertia penetrates the fabric, while without inertia it collides with the side of the inter-yarn pore and is filtered out. Inertia carries a particle closer to the centre of an inter-yarn pore where it is further from the sides and so escapes colliding with these walls.

Small amounts of inertia reducing filtration efficiency is contrary to the standard picture of filtration<sup>5</sup> we outlined in the introduction. In that standard picture, deviations from streamlines due to inertia always increase the probability of a collision. At large Stokes number we indeed find that inertia increases filtration efficiency. However, here and in Robinson *et al.*<sup>9</sup> we find that at small Stokes numbers the situation can be more complex and subtle. It can make filtration a little less efficient.

Interception can be thought of as filtration in zero air speed U limit, as then the Stokes number is zero. Thus our finding of predominantly interception filtration implies that filtration by our fabric is almost independent of U, or equivalently of the pressure drop across the fabric. This is in agreement with findings of Konda  $et\ al.^{11}$  who found that filtration did not vary significantly when they varied the pressure drop across the sample.

# VII. FILTRATION VIA PARTICLES DIFFUSING INTO CONTACT

Filtration of particles of order 100 nm is typically dominated by diffusion of the particles onto the surfaces of the filter<sup>5</sup>. The nanoparticles then stick and are filtered out. With a flow field based on imaging at 1.8 µm resolution, we are unable to be quantitative about the filtration efficiency for particles in this size range. But we are able to argue that the efficiency of filtration by diffusion should be low. The argument is as follows.

For our fabric, almost all air flows through inter-yarn pores  $\sim 50\,\mu\text{m}$  across. So filtration by diffusion relies on a particle diffusing across the flowing air stream into contact with the sides of the inter-yarn pore, during the short time the particle is being advected through the fabric. So filtration efficiency is determined by the ratio of a diffusive time  $t_{DX}$ , to an advection time  $t_{A}$ .  $t_{DX}$  is the time taken to diffuse across (i.e., in xy plane) an inter-yarn pore.  $t_{A}$  is the time taken for air to flow through

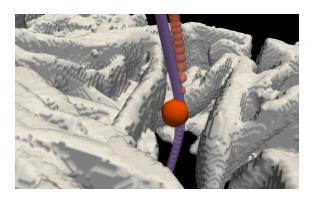


FIG. 11. A pair of trajectories with and without inertia, that start at the same point. This is for a particle of diameter  $20\,\mu m$ . The fabric is shown in white, and trajectories with and without inertia are traced out by purple and by orange spheres, respectively. The sphere at the collision point is shown at the true particle size, others along the path are smaller, for clarity. Note that with inertia the particle penetrates the fabric while without it, the particle collides at the point shown by the large orange sphere. Here inertia carries the particle a little farther out from the side of the inter-yarn pore, avoiding a collision. Image produced with ParaView  $^{42}$ .

the pore.

The ratios of diffusive to flow timescales are called Péclet numbers. Here the Péclet number is

$$Pe = \frac{t_{DX}}{t_A} \tag{15}$$

For a particle 100 nm in diameter, Stokes-Einstein gives  $D = kT/(3\pi\mu d_p) \sim 240\,\mu\text{m}^2\,\text{s}^{-1}$ , and so for a distance of 50  $\mu$ m,  $t_{DX} \sim (50^2)/80 \sim 10\,\text{s}$ . The advection timescale is just the time taken for air to flow through the fabric  $t_A \sim 100\,\mu\text{m}/2.7\,\text{cm}\,\text{s}^{-1} \sim 4\,\text{ms}$ . Thus

$$Pe \sim 3000$$
 (16)

As Pe  $\gg 1$  then particles with  $d_p = 300\,\mathrm{nm}$  are carried through the fabric much faster than they can diffuse across the interyarn pores, and we expect the efficiency of filtration by diffusion to be very low. Note that for larger particles D is smaller so filtration by diffusion is even less efficient.

# VIII. CONCLUSION

We predict that the filtration efficiency of our imaged fabric is approximately 5%. This is for particles of diameter 1.5 µm, which is around the most probable size for droplets exhaled while speaking <sup>19</sup>. The filtration efficiency will be even lower for these droplets once they have <sup>19</sup> entered room air, and evaporation has halved their diameter to less than a micrometre. Our filtration efficiency is for approximately half a layer of woven fabric with an estimated TPI of 186. Konda *et al.* <sup>11</sup> found filtration efficiencies of 9% and 18% for (complete single layers of) woven fabrics of 80 and 160 TPI. Sankhyan *et al.* <sup>13</sup> found similar values. As we study only approximately

half a layer, it is difficult to make quantitative comparisons, but Konda *et al.*, Sankhyan *et al.* and this work all find that the filtration efficiency of a single layer of woven fabrics is very low.

We have the complete flow field (at a resolution of  $1.8\,\mu m$ ) inside the fabric, and we can also control the inertia of the particles, so we can see why the efficiency is so low. The efficiency is low because essentially all the air flows through relatively large (tens of micrometres) inter-yarn pores, which are only obstructed by a few stray fibres, see Fig. 5. Particles just follow the air through these gaps and so few are filtered out.

Filtration is due to interception over the size range from one to a few tens of micrometres we study. Surprisingly, over this size range, the effect of inertia is to decrease filtration efficiencies, although the effect is small. Modest amounts of inertia decrease filtration efficiency by pushing more particle trajectories away from collisions with fibres, than they do trajectories towards collisions. Very large amounts of inertia (for example due to a sneeze greatly increasing U) will increase efficiency due to most of the fabric area being occupied by yarns. Diffusion filtration efficiency for droplets in this size range is inefficient<sup>5</sup>, see section VII.

So, we conclude that at the droplet size we want to filter, all mechanisms are inefficient, and this is a consequence of the fact that the woven fabric has little structure on lengthscales below the inter-yarn pore size of tens of micrometres. This is in contrast to the non-woven filters in surgical masks and respirators (such as the European standard FFP and American standard N95 respirators), which force the air around single fibres of typical size around 15 µm. This smaller lengthscale in surgical masks brings inertial filtration into play for droplets around a few micrometres in diameter<sup>9</sup>. The (large) yarn lengthscale is a fundamental feature of all woven fabrics, which are woven from these yarns. So it may be that woven fabrics are inherently less efficient filters than non-woven filters.

### A. Limitations of the present work, and future work

We have simulated the flow field through one sample of woven fabric at a resolution of  $1.8\,\mu m$ , and used this to understand the observed poor filtration performance. Future work could look at different fabrics, with different TPIs, and go to higher resolution. Higher resolution images will improve the estimation of the filtration of smaller particles in particular, as this is likely to be sensitive to yarn/fibre roughness of length-scales of a micrometre and smaller.

Although our fabric is a poor filter, some data suggests that fabrics with higher TPIs are better filters<sup>11</sup>, possibly because the inter-yarn pores are smaller. Filtration can also be improved by using multiple layers<sup>13</sup>. However, both multiple layers and higher TPI lead to higher impedance to air flow. Filtration is always a trade off between maximising the fraction of particles filtered, while keeping the impedance of the filter to air flow within acceptable limits. Future work should consider whether woven materials are inherently bet-

ter or worse than non-woven materials, at achieving a high filtration to impedance ratio.

### **ACKNOWLEDGMENTS**

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### DATA AVAILABILITY STATEMENT

The data and computer code that support the findings of this study are openly available in Zenodo at http://doi.org/10.5281/zenodo.5552357.

# Appendix A: Computational details for integrating particle trajectories

Each particle trajectory is obtained by starting the particle at z=5 in LB units, and at x and y coordinates on a square grid in the central quarter of the box, i.e., from  $n_x/4$  to  $3n_x/4$  along the x axis and from  $n_y/4$  to  $3n_y/4$  along the y axis. We varied the starting region for the trajectories to observe the dependence of efficiency on starting region, and the efficiency varied by amounts around 10%. The particle starts with the same velocity as the local flow velocity. Weighting the trajectories by their initial velocities using Eq. (14) makes a difference of approximately twenty percent for our box with 200 LB lattice spacings in front of the fabric. It makes more of a difference for shorter boxes along z, hence our box size is trade off between accuracy and computational cost.

Of order 20% of trajectories leave the box at the sides. These are not counted in our flux calculations. Although the LB flow field has periodic boundary conditions at the sides, this does not reproduce well the true conditions in the fabric, which is not perfectly periodic in x and y. We cannot use a larger box along x and y because the larger box reaches the edge of the fabric strip, and a defect in the fabric, see Fig. 2.

So we have multiple sources of uncertainties, each ten or a few tens of percent. Plus we only couple the particle to the fluid flow at the particle's centre of mass, and are using a flow field with spatial resolution larger than the smallest particles we consider. Considering all these sources of uncertainty, and the approximations of the model, we estimate that our results are accurate to about a factor of two.

Each trajectory is integrated forward in time, using adaptive-step-size modified Euler integration of Eq. (11), until the particle either collides with a fibre voxel, or reaches the bottom (large z) edge of the simulation box. At each time

step, we check for a collision. A collision occurs if the centre of the particle is within a distance  $(1/2)(d_p + \delta)$ , i.e., radius of the particle plus a correction  $\delta$ . We estimate that the optimal value of  $\delta$  is 0.5 in LB units. So we use this value throughout this work.

The integration of Eq. (11) requires we determine  $t_I$  in LB units. This is done as follows, for the example of a particle with  $d_p = 5 \,\mu\text{m}$ . First, we obtain the mean velocity in the LB flow field in a slice far from the fabric, as  $U = 5.8 \times 10^{-7}$  in LB units. Second, we use Eq. (13) to determine that St = 1.16, for lengthscale  $L = 1.8 \,\mu\text{m}$  and  $U = 2.7 \,\text{cm s}^{-1}$ . Then we use Eq. (12) in LB units to obtain, with L = 1 and our LB U, that  $t_I = 20.8 \times 10^6$  in LB units. This value of  $t_I$  reproduces the correct Stokes number for a particle  $5 \,\mu\text{m}$  in diameter. The particle then collides with any lattice site within a distance of 1.89 LB units.

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