Superluminal Communication?

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1 Abstract

A possible superluminal communication method is discussed. The method is based on a modified version of the famous delayed choice quantum eraser (DCQE) experiment by Kim et al.[1]. An alternative and step-by-step analysis for the DCQE is discussed, which will be used later to analyze the modified setup.

2 Revision of the delayed choice quantum eraser experiment by Kim et al.

This famous experiment is taken by many people as evidence of retrocausality. However, it can actually be explained without invoking retrocausality.

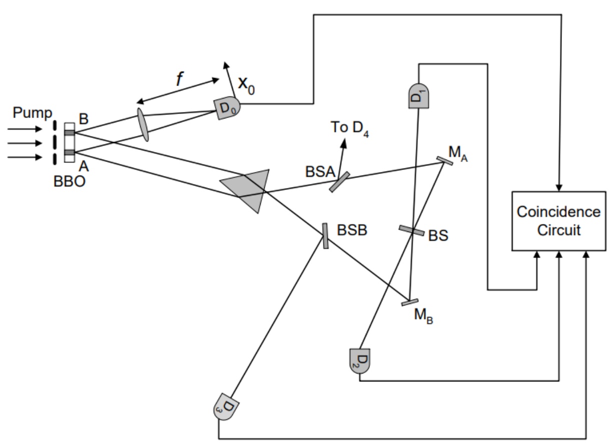

Figure 1: A copy of the schematic diagram of the experimental setup in the paper [1]. The pump laser beam is divided by a double slit and forms two regions A and B inside the BBO crystal. A pair of signal-idler photons is then generated from either the A or the B region.

Standard photon detection quantum mechanical calculation is presented in [1]. They will be briefly stated here.

The joint detection counting rate, R_{0j}, of detector D_0 and detector D_j, in the time interval T is:

$$\begin{aligned} R_{0j} &\propto \frac{1}{T}\int_0^T\int_0^T dT_0 dT_j \langle\Psi|E_0^{(-)}E_j^{(-)}E_j^{(+)}E_0^{(+)}|\Psi\rangle \\ &= \frac{1}{T}\int_0^T\int_0^T dT_0 dT_j \langle\Psi|E_0^{(-)}E_j^{(-)}|0\rangle\langle 0|E_j^{(+)}E_0^{(+)}|\Psi\rangle \\ &= \frac{1}{T}\int_0^T\int_0^T dT_0 dT_j |\langle 0|E_j^{(+)}E_0^{(+)}|\Psi\rangle|^2 \end{aligned} \tag{1}$$

A second order correlation function is used because two photon detectors are involved in the joint detection. $|\Psi\rangle$ is the spontaneous parametric down converted (SPDC) entangled state:

$$|\Psi\rangle = \sum_{s,i} C(\mathbf{k}_s, \mathbf{k}_i) a_s^\dagger[\omega(\mathbf{k}_s)] a_i^\dagger[\omega(\mathbf{k}_i)]|0\rangle \tag{2}$$

where:

$$C(\mathbf{k}_s, \mathbf{k}_i) = \delta(\omega_s + \omega_i - \omega_p)\delta(\mathbf{k}_s + \mathbf{k}_i - \mathbf{k}_p) \tag{3}$$

which represents energy and momentum conservation, and also the corresponding entanglement at the same time. The absolute square of the second order correlation function can be rewritten as:

$$\Psi(t_0, t_j) \equiv |\langle 0|E_j^{(+)}E_0^{(+)}|\Psi\rangle|^2 \tag{4}$$

and is called the joint count probability amplitude. Four different scenarios can happen, corresponding to the four detectors set-up to detect the idler photon. They can be represented as the four different forms of $\Psi(t_0, t_j)$:

$$\begin{aligned} \Psi(t_0, t_1) &= A(t_0, t_1^A) + A(t_0, t_1^B) \\ \Psi(t_0, t_2) &= A(t_0, t_2^A) - A(t_0, t_2^B) \\ \Psi(t_0, t_3) &= A(t_0, t_3^A) \\ \Psi(t_0, t_4) &= A(t_0, t_4^B) \end{aligned} \tag{5}$$

The different sign in $\Psi(t_0, t_1)$ and $\Psi(t_0, t_2)$ are due to the unitary transformation from the beam splitter.

3 Alternative mathematical description

I would like to present an alternative form of the mathematical description to get more insight on the system.

3.1 After passing through the double slits

We assume that the probability for the photon to pass through is the same for both slits (A and B). Therefore, we can write the photon state as:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|A\rangle + |B\rangle) \tag{6}$$

3.2 After down conversion by BBO

The entanglements in Eq.(2) and (3) are present due to the conservation laws. We can represent the conservation laws in a different form:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|A\rangle + |B\rangle) \xrightarrow{\text{SPDC}} |\Psi\rangle = \frac{1}{\sqrt{2}}(|A,\mathbf{k}_s\rangle|A,\mathbf{k}_i\rangle + |B,\mathbf{k}_s\rangle|B,\mathbf{k}_i\rangle) \tag{7}$$

where $\mathbf{k}_s(\omega_s) + \mathbf{k}_i(\omega_i) = \mathbf{k}_p(\omega_p)$, and $\omega_s + \omega_i = \omega_p$.[1] The position space wavefunction (i.e. the probability amplitude) of the signal photon along the x-direction can be written as:

$$\begin{aligned} \Psi_s(x) = \langle x_s|\Psi\rangle &= \frac{1}{\sqrt{2}}(\langle x_s|A,\mathbf{k}_s\rangle|A,\mathbf{k}_i\rangle + \langle x_s|B,\mathbf{k}_s\rangle|B,\mathbf{k}_i\rangle) \\ &= \frac{1}{\sqrt{2}}(\Psi_s^A(x)|A,\mathbf{k}_i\rangle + \Psi_s^B(x)|B,\mathbf{k}_i\rangle) \end{aligned} \tag{8}$$

3.3 When the signal photon is detected by detector D_0

The probability distribution of the signal photon on D_0 is the absolute square of the wavefunction:

$$\begin{aligned} p_0(x) = |\Psi_s(x)|^2 &= \frac{1}{2}|(\Psi_s^A(x)|A,\mathbf{k}_i\rangle + \Psi_s^B(x)|B,\mathbf{k}_i\rangle)|^2 \\ &= \frac{1}{2}|\Psi_s^A(x)|^2 + |\Psi_s^B(x)|^2 \end{aligned} \tag{9}$$

There is no cross term ($\Psi_s^A\Psi_s^B$) due to the orthogonality between $|A,\mathbf{k}_i\rangle$ and $|B,\mathbf{k}_i\rangle$.

The physical meaning of this is: the "information of which slit the original

[1] The SPDC transformation by the BBO crystal is not discussed here.

photon went through" (a.k.a the "which-slit-information") is also carried by the idler photon, and since the available paths to be travelled by the idler photon are well separated and distinguishable, one can in principle find out the which-slit-information by performing some measurement on the idler photon. This availability of the information acts like a distinguishable label to the signal photon, making it unable to produce interference pattern.

The probability that the signal photon is detected at x_0 with the uncertainty Δ is:

$$P_0\left(x_0 \pm \frac{\Delta}{2}\right) = \int_{x_0-\frac{\Delta}{2}}^{x_0+\frac{\Delta}{2}} p_0(x)dx' \tag{10}$$

The experimental setup is designed such that $\Psi_s^A(x)$ and $\Psi_s^B(x)$ overlaps sufficiently to allow interference to happen when it is possible. When the signal photon is detected and absorbed (or destroyed), the photons pair is entangled to the measuring device, and due to the macroscopic size of the measuring device and our inability to track how the phase change, decoherence happens, and using the terminology of the Copenhagen interpretation, the state of the photon pairs partially **collapses** into:

$$\begin{aligned} |\Psi\rangle &= \frac{1}{\sqrt{2}}(|A,\mathbf{k}_s\rangle|A,\mathbf{k}_i\rangle + |B,\mathbf{k}_s\rangle|B,\mathbf{k}_i\rangle) \\ \xrightarrow{\text{partial collapse}} |\Psi\rangle &= \epsilon(\Psi_s^A(x_0)|A,\mathbf{k}_i\rangle + \Psi_s^B(x_0)|B,\mathbf{k}_i\rangle) \\ &\equiv \alpha|A\rangle_i + \beta|B\rangle_i \end{aligned} \tag{11}$$

where ϵ is a re-normalization factor:

$$\epsilon = \frac{1}{\sqrt{|\Psi_{s0}^A|^2 + |\Psi_{s0}^B|^2}} \tag{12}$$

Since $\Psi_s^A(x)$ and $\Psi_s^B(x)$ overlap, we are unable to determine from which slit the original photon went through just by knowing where the signal photon landed on D_0. Therefore, the state of the idler photon[2] is still a superposition state, and I called this as a partial collapse.

We will see later that α and β must be complex numbers, which implies that they are not just square root of the values of the probability distribution but they are probability amplitudes, and this further supports that the collapse is only partial.

If we use the 'collapse'-picture of the Copenhagen interpretation, the collapse process is not time reversible, and therefore not a unitary process, i.e. probability is not conserved. Therefore, a re-normalization factor is needed.

[2]Since the signal photon is destroyed, the new state is the updated state of just the idler photon.

3.4 If the idler photon is detected by detector D_3

If the idler photon is detected by detector D_3, the state of the idler photon $|\Psi\rangle$ collapses into[3]:

$$|\Psi\rangle = \alpha|A\rangle_i + \beta|B\rangle_i \xrightarrow{\text{collapse}} |\Psi\rangle = |D_3\rangle \tag{13}$$

which tells us that the original photon "went through" slit B.[4] The probability that the idler photon will be detected by D_3 is:

$$P_{03} = \int_{x_0-\frac{\Delta}{2}}^{x_0+\frac{\Delta}{2}} |\epsilon\Psi_s^B(x)|^2 dx' \tag{14}$$

where Δ is the uncertainty of the landing position of the signal photon on detector D_0.

3.5 If the idler photon is detected by detector D_4

If the idler photon is detected by detector D_4, the state of the idler photon $|\Psi\rangle$ collapses into[5]:

$$|\Psi\rangle = \alpha|A\rangle_i + \beta|B\rangle_i \xrightarrow{\text{collapse}} |\Psi\rangle = |D_4\rangle \tag{15}$$

which tells us that the original photon "went through" slit A. The probability that the idler photon will be detected by D_4 is:

$$P_{04} = \int_{x_0-\frac{\Delta}{2}}^{x_0+\frac{\Delta}{2}} |\epsilon\Psi_s^A(x)|^2 dx' \tag{16}$$

where Δ is the uncertainty of the landing position of the signal photon on detector D_0.

[3]The state after the collapse is the state for the detector D_3, and not the state of the idler photon, because the idler photon is destroyed when it is detected.

[4]Actually, I think, when we use the words "from which slit the photon went through", it seems to imply that we are suggesting realism. But here I use these words just because I have not found a better alternative and just out of convenience. I am not suggesting realism.

[5]The state after the collapse is the state for the detector D_4, and not the state of the idler photon, because the idler photon is destroyed when it is detected.

3.6 If the idler photon is detected by detector D_1

If the idler photon is not sent towards D_3 and D_4, the state of the idler photon before it reaches the beam splitter can be written as:

$$|\Psi\rangle = \alpha|A\rangle + \beta|B\rangle \equiv \alpha|\mathrm{I}\rangle + \beta|\mathrm{II}\rangle \equiv \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \tag{17}$$

where I and II represent the two sides of the beam splitter. The effect of the beam splitter can be written as[6]:

$$B = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \tag{18}$$

which turns:

$$|\mathrm{I}\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad B|\mathrm{I}\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{19}$$

and[7]:

$$|\mathrm{II}\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad B|\mathrm{II}\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{20}$$

So,

$$B|\Psi\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \alpha + \beta \\ \alpha - \beta \end{pmatrix} = |\Psi'\rangle \tag{21}$$

The probability that the idler photon get detected by D_1 is:

$$\begin{aligned} P_{01}(x_0) = |\langle \mathrm{I}|\Psi'\rangle|^2 &= \frac{1}{2}|(\alpha+\beta)|^2 \\ &= \frac{1}{2}(|\alpha|^2 + |\beta|^2 + \alpha^*\beta + \beta^*\alpha) \\ &= \frac{1}{2}(|\alpha|^2 + |\beta|^2 + |\alpha||\beta|(e^{i\varphi_B - i\varphi_A} + e^{i\varphi_A - i\varphi_B})) \\ &= \frac{1}{2}(|\alpha|^2 + |\beta|^2 - 2|\alpha||\beta|\cos\varphi) \end{aligned} \tag{22}$$

If α and β are just the square root of the values in the probability distribution, i.e.

$$\alpha \propto \sqrt{|\Psi_s^A(x_0)|^2} \quad \text{and} \quad \beta \propto \sqrt{|\Psi_s^B(x_0)|^2}$$

and are not values in the probability amplitude, i.e.

$$\alpha \propto \Psi_s^A(x_0) \quad \text{and} \quad \beta \propto \Psi_s^B(x_0)$$

[6] This is also the matrix representation of a Hadamard gate.

[7] The minus sign in the B matrix is to keep the unitarity of the transformation

then α and β must be real numbers, and φ will always be zero, and there will be no interference pattern after we group the landing positions of the signal photons on D_0 according to where the idler photons get detected. Therefore, they must be complex numbers and $\varphi = \varphi(x_0)$, and this supports that the state has only partially collapsed after the signal photon get detected.

3.7 If the idler photon is detected by detector D_2

Similarly, if the idler photon is not sent towards D_3 and D_4, and later get detected by D_2, the probability that the idler photon get detected by D_2 is:

$$\begin{aligned} P_{02}(x_0) = |\langle \mathrm{II}|\Psi'\rangle|^2 &= \frac{1}{2}|(\alpha - \beta)|^2 \\ &= \frac{1}{2}(|\alpha|^2 + |\beta|^2 - \alpha^*\beta - \beta^*\alpha) \\ &= \frac{1}{2}(|\alpha|^2 + |\beta|^2 - |\alpha||\beta|(e^{i\varphi_B - i\varphi_A} + e^{i\varphi_A - i\varphi_B})) \\ &= \frac{1}{2}(|\alpha|^2 + |\beta|^2 - 2|\alpha||\beta|\cos\varphi) \end{aligned} \tag{23}$$

The difference in sign between the two probabilities is equivalent to a π-phase shift between the two interference patterns that arise after the landing positions of the signal photons on D_0 are grouped accordingly.

3.8 Conclusion

It is possible to describe the experiment result without having to invoke retro-causality. And one will never see interference pattern directly on D_0. The interference patterns are due to the grouping of the landing positions of the signal photons according to the detectors where the idler photons are detected.

4 Modification to the DCQE experiment

Assuming that all the explanation above are correct, let us see if we can modify the setup to allow superluminal communication.

Suppose that the signal photons are sent to someone called Alice, and the idler photons are sent to someone called Bob. Alice and Bob both have a clock and they are synchronized. Bob has a large detector D_0 that can resolve the landing position of the idler photons. The idler photons need a longer time to reach D_0 than the time needed for the signal photons to reach Alice's setup.

The state of the photon after passing through the slits and the state of the SPDC photons pair are the same as in Eq.(6) and Eq.(7) respectively.

4.1 If Alice chooses to find out the "which-slit-information"

Alice can choose to find out the "which-slit-information" by directing the signal photon to a detection system like D_3 and D_4, before the idler photon has enough time to reach Bob's detector D_0. Let's say that the signal photon reaches Alice's detectors at time $t = t_1$.

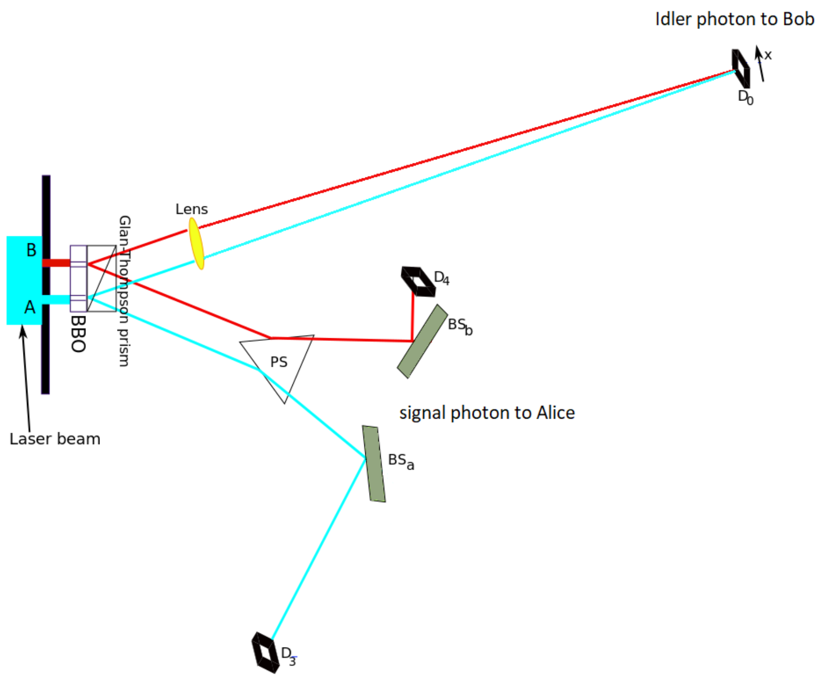

Figure 2: Schematic diagram of the modified setup when Alice chooses to find out the "which-slit-information" by directing the signal photons to detectors D_3 and D_4.

and the state of the photons pair collapses into either[8]:

$$\begin{aligned} |\Psi\rangle &= \frac{1}{\sqrt{2}}(|A, \mathbf{k}_s\rangle|A, \mathbf{k}_i\rangle + |B, \mathbf{k}_s\rangle|B, \mathbf{k}_i\rangle) \\ &\equiv \frac{1}{\sqrt{2}}(|D_3\rangle|A, \mathbf{k}_i\rangle + |D_4\rangle|B, \mathbf{k}_i\rangle) \qquad (24) \\ \xrightarrow{\text{collapse}} |\Psi'\rangle &= |A, \mathbf{k}_i\rangle \equiv |A\rangle_i \end{aligned}$$

when the signal photon is detected by D_3, with the probability:

[8]The state after the collapse is the state just for the idler photon because the signal photon is absorbed and destroyed, and we also ignore the state of the environment.

$$P_3 = |\langle D_3|\Psi\rangle|^2 = \frac{1}{2} \tag{25}$$

or into:

$$\begin{aligned} |\Psi\rangle &\equiv \frac{1}{\sqrt{2}}(|D_3\rangle|A,\mathbf{k}_i\rangle + |D_4\rangle|B,\mathbf{k}_i\rangle) \\ &\xrightarrow{\text{collapse}} |\Psi'\rangle = |B,\mathbf{k}_i\rangle \equiv |B\rangle_i \end{aligned} \tag{26}$$

when the signal photon is detected by D_4, with the probability:

$$P_4 = |\langle D_4|\Psi\rangle|^2 = \frac{1}{2} \tag{27}$$

Alice can continue doing this for many of her signal photons for a duration τ, such that there will be enough idler photons on Bob's side to form some distribution patterns.

Let's say at time $t = t_2 = t_1 + \delta t$, the idler photon reaches Bob's detector D_0, and he will start recording the landing positions of the idler photons. He will check if any pattern forms on his detector D_0 after a duration τ.

How will the distribution looks like on Bob's detector if Alice choose to find out the "which-slit-information"? The distribution along the x-axis can be calculated from the absolute square of the wave function of Bob's idler photon.

If the idler photon's entangled signal photon is detected by D_3, the probability distribution of that idler photon is:

$$p_{30}(x) = |\langle x|A\rangle_i|^2 = |\Psi_i^A(x)|^2 \tag{28}$$

If the idler photon's entangled signal photon is detected by D_4, the probability distribution of that idler photon is:

$$p_{40}(x) = |\langle x|B\rangle_i|^2 = |\Psi_i^B(x)|^2 \tag{29}$$

Since Alice did not tell Bob about the results of her measurements, and the setup on Bob's side is designed in such a way that $\Psi_i^A(x)$ overlaps with $\Psi_i^B(x)$, Bob cannot separate the distributions. Therefore, the resultant distribution that he sees on D_0 is the combination of the two distributions:

$$p_0(x) = \frac{1}{2}\cdot p_{30}(x) + \frac{1}{2}\cdot p_{40}(x) = \frac{1}{2}(|\Psi_i^A(x)|^2 + |\Psi_i^B(x)|^2) \tag{30}$$

The factor $\frac{1}{2}$ is due to the probabilities in Eq.(25) and (27). As expected, there is no term that corresponds to interference pattern, and Bob will not see an interference pattern, because the state of the idler photon is no longer a superposition state after Alice has "found out the which-slit-information".

4.2 If Alice chooses not to find out the "which-slit-information"

If Alice choose to not find out the which-slit-information, and does nothing to the signal photons, the superposition state of the photons pair remains. The wavefunction of the idler photon is:

$$\begin{aligned}\Psi_i(x) = \langle x_i|\Psi\rangle &= \frac{1}{\sqrt{2}}(|A,\mathbf{k}_s\rangle\langle x_i|A,\mathbf{k}_i\rangle + |B,\mathbf{k}_s\rangle\langle x_i|B,\mathbf{k}_i\rangle) \\ &= \frac{1}{\sqrt{2}}(\Psi_i^A(x)|A,\mathbf{k}_s\rangle + \Psi_i^B(x)|B,\mathbf{k}_s\rangle)\end{aligned} \tag{31}$$

In this case, will Bob be able to see an interference pattern on D_0? The probability distribution of his idler photon on D_0 is again the absolute square of the idler photon's wavefunction:

$$p_0(x) = |\Psi_i(x)|^2 = \frac{1}{2}(|\Psi_i^A(x)|^2 + |\Psi_i^B(x)|^2) \tag{32}$$

Again, there is no term that corresponds to interference, and the distribution is identical to Eq.(30). Therefore, Bob will not see any interference pattern. The reason is the same reason why we cannot see an interference pattern on D_0 in the DCQE experiment by Kim et al, which is due to the orthogonality between $|A,\mathbf{k}_s\rangle$ and $|B,\mathbf{k}_s\rangle$.

Is there anything Alice can do to fix this? Maybe Alice can "erase"[9] the which-slit-information contained in the signal photon before the idler photon get detected at Bob's detector. If Alice uses the "quantum eraser" in DCQE experiment by Kim et al., will we be able to see interference pattern on Bob's detector?

[9] Maybe the better word is "obscure", because the word "erase" sounds like we are going to hit the law of conservation of quantum information head-on.

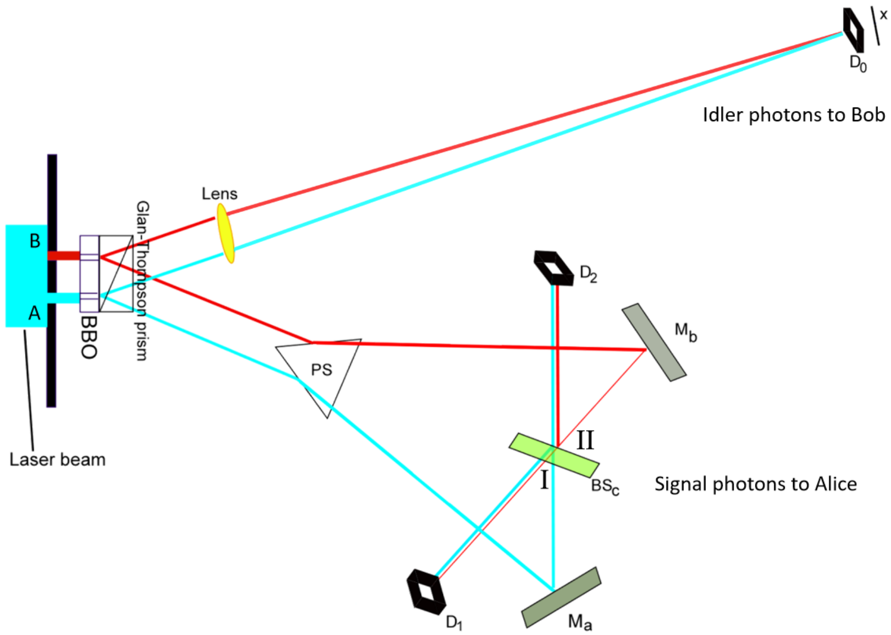

Figure 3: Schematic diagram of the quantum eraser setup that is similar to the one used in Kim et al.'s DCQE experiment.

The state of the photons pair before the signal photon reaches the 50-50 beam splitter can be written as:

$$
\begin{aligned}
|\Psi\rangle &= \frac{1}{\sqrt{2}}(|A,\mathbf{k}_s\rangle|A,\mathbf{k}_i\rangle + |B,\mathbf{k}_s\rangle|B,\mathbf{k}_i\rangle) \\
&\equiv \frac{1}{\sqrt{2}}(|\mathrm{I}\rangle_s|A\rangle_i + |\mathrm{II}\rangle_s|B\rangle_i) \equiv \frac{1}{\sqrt{2}}\begin{pmatrix}|A\rangle_i \\ |B\rangle_i\end{pmatrix}
\end{aligned}
\tag{33}
$$

When the signal photon leaves the beam splitter, the state of the photon pairs become:

$$
\begin{aligned}
|\Psi'\rangle = B|\Psi\rangle &= \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1 \\ 1 & -1\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}|A\rangle_i \\ |B\rangle_i\end{pmatrix} \\
&= \frac{1}{2}\begin{pmatrix}|A\rangle_i + |B\rangle_i \\ |A\rangle_i - |B\rangle_i\end{pmatrix} \\
&= \frac{1}{2}\Big[(|A\rangle_i + |B\rangle_i)|\mathrm{I}\rangle_s + (|A\rangle_i - |B\rangle_i)|\mathrm{II}\rangle_s\Big]
\end{aligned}
\tag{34}
$$

If the signal photon is detected by D_1, then the state of the photons pair partially collapses into[10]:

$$
\begin{aligned}
&|\Psi'\rangle = \frac{1}{2}\Big[(|A\rangle_i + |B\rangle_i)|\mathrm{I}\rangle_s + (|A\rangle_i - |B\rangle_i)|\mathrm{II}\rangle_s\Big] \\
\xrightarrow{\text{partial collapse}} &|\Psi'\rangle = \eta\Big[|A\rangle_i + |B\rangle_i\Big]
\end{aligned}
\tag{35}
$$

with the normalization factor $\eta = \frac{1}{\sqrt{2}}$. The wavefunction of the idler photon is:

$$
\Psi_i(x) = \langle x|\Psi'\rangle = \frac{1}{\sqrt{2}}(\langle x|A\rangle_i + \langle x|B\rangle_i) = \frac{1}{\sqrt{2}}(\Psi_i^A(x) + \Psi_i^B(x)) \tag{36}
$$

The probability distribution of the landing position of the idler photon on Bob's D_0 is the absolute square of the idler photon's wavefunction:

$$
p_{10}(x) = |\Psi_i(x)|^2 = \frac{1}{2}(|\Psi_i^A(x)|^2 + |\Psi_i^B(x)|^2 + 2|\Psi_i^A(x)||\Psi_i^B(x)|\cos\varphi) \tag{37}
$$

If the signal photon is detected by D_2, then the state of the photons pair partially collapses into:

$$
\begin{aligned}
&|\Psi'\rangle = \frac{1}{2}\Big[(|A\rangle_i + |B\rangle_i)|\mathrm{I}\rangle_s + (|A\rangle_i - |B\rangle_i)|\mathrm{II}\rangle_s\Big] \\
\xrightarrow{\text{partial collapse}} &|\Psi'\rangle = \eta\Big[|A\rangle_i - |B\rangle_i\Big]
\end{aligned}
\tag{38}
$$

with the normalization factor $\eta = \frac{1}{\sqrt{2}}$. The wavefunction of the idler photon is:

$$
\Psi_i(x) = \langle x|\Psi'\rangle = \frac{1}{\sqrt{2}}(\langle x|A\rangle_i - \langle x|B\rangle_i) = \frac{1}{\sqrt{2}}(\Psi_i^A(x) - \Psi_i^B(x)) \tag{39}
$$

The probability distribution of the landing position of the idler photon on Bob's D_0 is the absolute square of the idler photon's wavefunction:

$$
p_{20}(x) = |\Psi_i(x)|^2 = \frac{1}{2}(|\Psi_i^A(x)|^2 + |\Psi_i^B(x)|^2 - 2|\Psi_i^A(x)||\Psi_i^B(x)|\cos\varphi) \tag{40}
$$

Interference terms are present in both Eq.(37) and (40), however, they have different signs. Alice's signal photons are equally likely to be detected by D_1 or D_2 if the beam splitter is a 50-50 beam splitter.

Therefore, the distribution that Bob sees is the combination of both distributions:

$$
p_0(x) = \frac{1}{2}\cdot p_{10}(x) + \frac{1}{2}\cdot p_{20}(x) = \frac{1}{2}(|\Psi_i^A(x)|^2 + |\Psi_i^B(x)|^2) \tag{41}
$$

[10]The state after the partial collapse is the state just for the idler photon, as the signal photon is annihilated after detection.

where the interference patterns of the two distributions cancel out, and we get back the same distribution as in Eq.(30) and (32), which means Bob is still unable to see any interference pattern and not able to tell what Alice has done to her signal photons by just looking at his idler photons.

4.3 Other way to obscure the "which-slit-information"

What if Alice just "erases" the "which-slit-information" by combining the two possible paths of the signal photon into a single path?

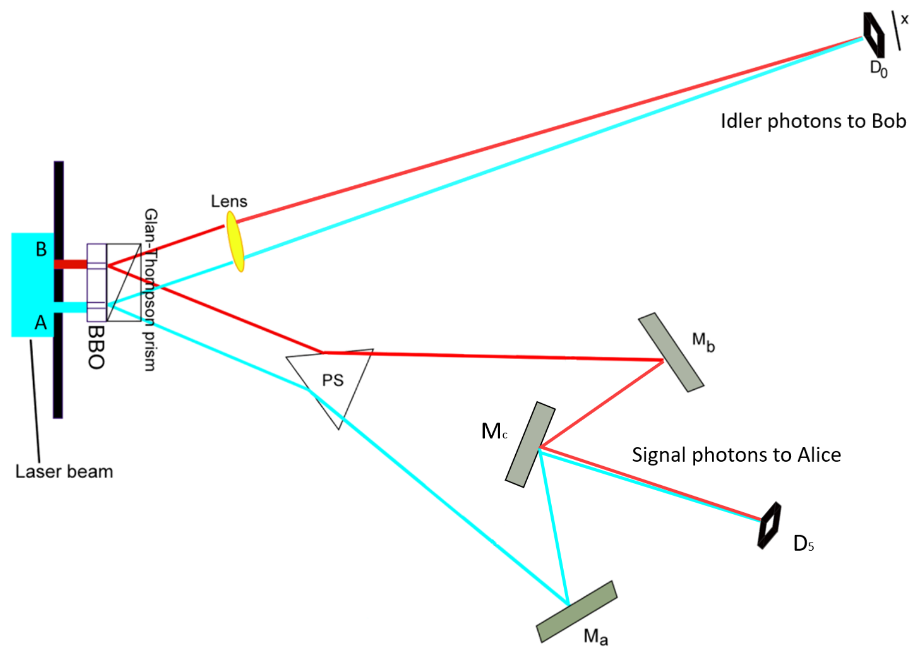

Figure 4: Schematic diagram of the setup to combine the paths of the signal photons into a single path

After the combination, Alice seems to have no way to find out the which-slit-information from the signal photons. Alice can move detector D_5 and use optical elements such as parabolic mirrors and lenses to make the focus point on D_5 as small as possible. Assuming that the state of the photons pair remains its form after the path combination, the wavefunction of the signal photon along the u-axis can be written as:

$$\begin{aligned}\Psi_s(u) = \langle u_s|\Psi\rangle &= \frac{1}{\sqrt{2}}(\langle u_s|A,\mathbf{k}_s\rangle|A,\mathbf{k}_i\rangle + \langle u_s|B,\mathbf{k}_s\rangle|B,\mathbf{k}_i\rangle) \\ &= \frac{1}{\sqrt{2}}(\Psi_s^A(u)|A,\mathbf{k}_i\rangle + \Psi_s^B(u)|B,\mathbf{k}_i\rangle)\end{aligned} \tag{42}$$

The probability distribution of the signal photon on D_5 is again the absolute square of its wavefunction:

$$p_5(u) = |\Psi_s(u)|^2 = \frac{1}{2}(|\Psi_s^A(u)|^2 + |\Psi_s^B(u)|^2) \tag{43}$$

Like in the DCQE experiment by Kim et al, Alice will not see interference pattern on D_5 due to the orthogonality between the states of the idler photon. We assume Alice manages to make the focus point very small on D_5, with peak centered at position $u = u_p$. When the signal photon is detected by Alice's detector D_5, the state of the photons pair partially collapses into[11]:

$$\begin{aligned} |\Psi\rangle &= \frac{1}{\sqrt{2}}(|A, \mathbf{k}_s\rangle|A, \mathbf{k}_i\rangle + |B, \mathbf{k}_s\rangle|B, \mathbf{k}_i\rangle) \\ \xrightarrow{\text{partially collapse}} |\Psi\rangle &= \epsilon(\Psi_s^A(u_0)|A\rangle_i + \Psi_s^B(u_0)|B\rangle_i) \\ &= \alpha|A\rangle_i + \beta|B\rangle_i \end{aligned} \tag{44}$$

where ϵ is just a normalization factor.[12] Similar to the situation in the Kim et al.'s DCQE experiment, the state of the idler photon after the signal photon is detected is still a superposition state, because even in principle we cannot find out the "which-slit-information" just by knowing the distribution on D_5.

Side note:

The detector D_5 could also be just an atom inside a cavity. Regardless of the state of the signal photon, when the signal photon is absorbed by the atom, the energy state of the atom changes from a low energy state $|D_5, g\rangle$ to an excited state $|D_5, e\rangle$, and later decays back to the lower energy state via spontaneous emission, emitting the radiation in random direction $\mathbf{k}_{\text{random}}$, effectively erases the "which-path-information".[13]

$$\begin{aligned} |\Psi\rangle &= \frac{1}{\sqrt{2}}(|A\rangle_s|A\rangle_i + |B\rangle_s|B\rangle_i)|D_5, g\rangle \\ \to |\Psi'\rangle &= \frac{1}{\sqrt{2}}(|A\rangle_i + |B\rangle_i)|D_5, e\rangle \\ \to |\Psi''\rangle &= \frac{1}{\sqrt{2}}(|A\rangle_i + |B\rangle_i)|D_5, g\rangle|\mathbf{k}_{\text{random}}\rangle_\gamma \\ &= |\Psi\rangle_i|D_5, g\rangle|\mathbf{k}_{\text{random}}\rangle_\gamma \end{aligned} \tag{45}$$

[11] The state after the partial collapse is the state of the idler photon because the signal photon is absorbed and destroyed after detection.

[12] α and β are complex numbers. The argument is given previously in the discussion about the DCQE experiment.

[13] This seems to violate the conservation of quantum information, and I personally think that this is similar to the black-hole information paradox. However, I am not knowledgeable enough to verify this.

We can also rewrite this in occupation number representation:

$$
\begin{aligned}
|\Psi\rangle &= \frac{1}{\sqrt{2}}(|A\rangle_s|A\rangle_i + |B\rangle_s|B\rangle_i)|D_5, g\rangle \\
&\overset{\text{Fock}}{\equiv} \frac{1}{\sqrt{2}}(a_s^\dagger a_i^\dagger|0\rangle_A + a_s^\dagger a_i^\dagger|0\rangle_B)|D_5, g\rangle
\end{aligned} \tag{46}
$$

$$
\begin{aligned}
|\Psi'\rangle = T_1|\Psi\rangle = b^\dagger a_s|\Psi\rangle &= b^\dagger a_s \frac{1}{\sqrt{2}}(a_s^\dagger a_i^\dagger|0\rangle_A + a_s^\dagger a_i^\dagger|0\rangle_B)|D_5, g\rangle \\
&= b^\dagger \frac{1}{\sqrt{2}}(a_s a_s^\dagger a_i^\dagger|0\rangle_A + a_s a_s^\dagger a_i^\dagger|0\rangle_B)|D_5, g\rangle \\
&= b^\dagger \frac{1}{\sqrt{2}}(a_i^\dagger|0\rangle_A + a_i^\dagger|0\rangle_B)|D_5, g\rangle \\
&= \frac{1}{\sqrt{2}}(a_i^\dagger|0\rangle_A + a_i^\dagger|0\rangle_B)b^\dagger|D_5, g\rangle \\
&= \frac{1}{\sqrt{2}}(a_i^\dagger|0\rangle_A + a_i^\dagger|0\rangle_B)|D_5, e\rangle \\
&\equiv |\Psi\rangle_i|D_5, e\rangle
\end{aligned} \tag{47}
$$

$$
\begin{aligned}
|\Psi''\rangle = T_2|\Psi'\rangle|0\rangle = a^\dagger b|\Psi'\rangle|0\rangle &= a^\dagger b|\Psi\rangle_i|D_5, e\rangle|0\rangle \\
&= a^\dagger|\Psi\rangle_i b|D_5, e\rangle|0\rangle \\
&= a^\dagger|\Psi\rangle_i|D_5, g\rangle|0\rangle \\
&= |\Psi\rangle_i|D_5, g\rangle a^\dagger|0\rangle
\end{aligned} \tag{48}
$$

$$
|\Psi''\rangle = |\Psi\rangle_i|D_5, g\rangle|\mathbf{k}_{\text{random}}\rangle_\gamma \overset{\text{Fock}}{\equiv} |\Psi\rangle_i|D_5, g\rangle a^\dagger|0\rangle \tag{49}
$$

End of side note.

After the partial collapse, the wavefunction of the idler photon can be written as:

$$
\begin{aligned}
\Psi_i(x) = \langle x_i|\Psi\rangle_i &= \alpha\langle x_i|A\rangle_i + \beta\langle x_i|B\rangle_i \\
&= \alpha\Psi_i^A(x) + \beta\Psi_i^B(x) \\
&= \epsilon(\Psi_s^A(u_0)\Psi_i^A(x) + \Psi_s^B(u_0)\Psi_i^B(x)) \\
&= \epsilon(\Psi_{s0}^A\Psi_i^A(x) + \Psi_{s0}^B\Psi_i^B(x))
\end{aligned} \tag{50}
$$

The probability distribution of the idler photon on Bob's detector D_0 is:

$$
p_0(x) = |\Psi_i(x)|^2 = \epsilon^2\Big(|\Psi_{s0}^A\Psi_i^A|^2 + |\Psi_{s0}^B\Psi_i^B|^2 + 2|\Psi_{s0}^A\Psi_i^A||\Psi_{s0}^B\Psi_i^B|\cos\varphi\Big) \tag{51}
$$

where:

$$\begin{aligned}\varphi = \varphi_B - \varphi_A &= (\varphi_{B,s}(u_0) + \varphi_{B,i}(x)) - (\varphi_{A,s}(u_0) + \varphi_{A,i}(x)) \\ &= (\varphi_{B,s}(u_0) - \varphi_{A,s}(u_0)) + (\varphi_{B,i}(x) - \varphi_{A,i}(x)) \\ &= \varphi_s(u_0) + \varphi_i(x)\end{aligned} \tag{52}$$

I think that $\Psi^A_{s0}(u_0)$ and $\Psi^B_{s0}(u_0)$, and therefore also $\varphi_s(u_0)$, will be almost constant for all the signal photons, if the focus spot is sufficiently small:

$$u_0 = u_p + du \quad \text{and} \quad du \ll 1 \tag{53}$$

With this, it seems like we can now see interference pattern on Bob's detector D_0.

If this works, Alice can use this to send binary information to Bob, for example: Alice and Bob have agreed before the experiment that if Bob does not see an interference pattern on his detector, it means Alice sends him '0'; and if he sees an interference pattern, then it means Alice sends him a '1'.

When the separation between Alice and Bob is large enough, such that light sent by Alice will take a time longer than τ to reach Bob, this will become a superluminal communication, which can violate causality, as the order of cause and effect might be inverted for observers in some reference frames. Further more, this also violates the no-communication theorem or the no-signalling theorem.[14]

5 Summary and Personal request

A possible alternative explanation for the DCQE experiment without invoking retrocausality is given. After that, I proposed some modification to the setup to see if superluminal communication can be achieved. The mathematical framework that is used to explain the DCQE experiment is later used to predict the results after the modification.

With this setup, it seems like superluminal communication is possible and at the same time many important rules are violated. I hope someone can help me to find out the errors in this analysis and perhaps also carry out the experiment to verify everything.

References

[1] Y.H. Kim, R. Yu, S.P. Kulik, Y. Shih, and M.O. Scully, Delayed "Choice" Quantum Eraser, Phys. Rev. Lett. 84, 1, doi.

[14] I think maybe the non-unitarity of the collapse-process is the loophole.